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Engineering Department Modernization

WILLIAM S. CAMPBELL

The Storage of Energy in Some Activated Alkali Halide Phosphors

H. O. ALBRECHT AND C. E. MANDEVILLE

A New Electrodynamics

PARRY MOON AND DOMINA EBERLE SPENCER

On the Flow Behind an Axially Symmetrical Attached Curved Shock

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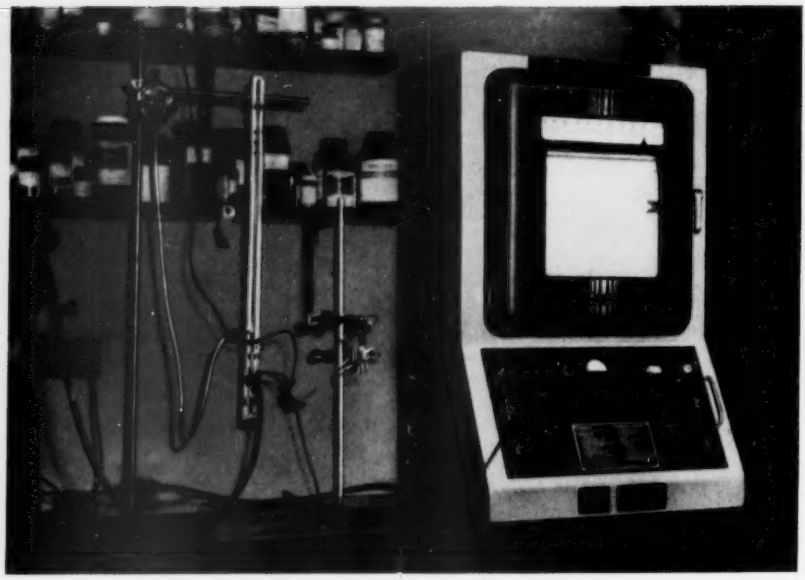

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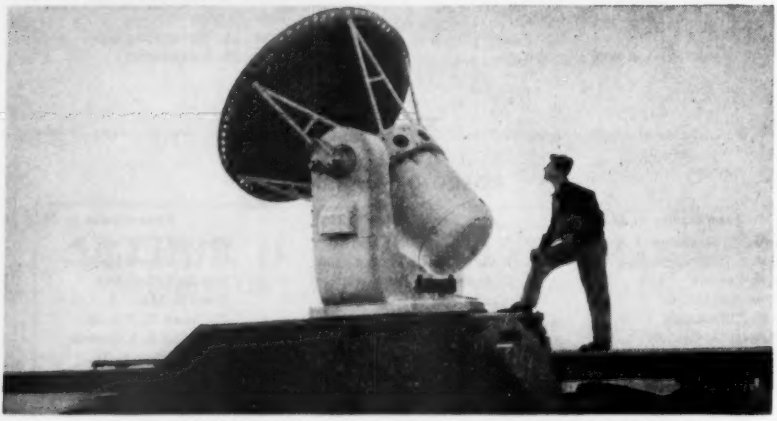

(Upper left) – Nike's missile climbs to destroy an enemy, under guidance of complex electronic controls. A radar is shown at right. Nike (pronounced Ny'kee) is named after the Greek goddess of Victory.

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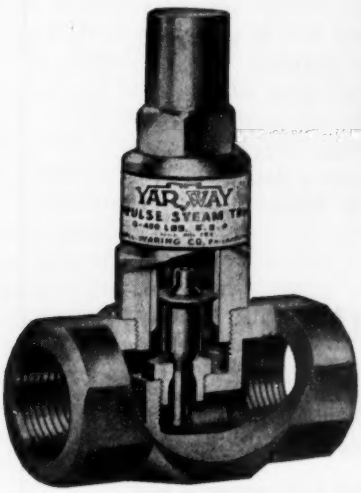

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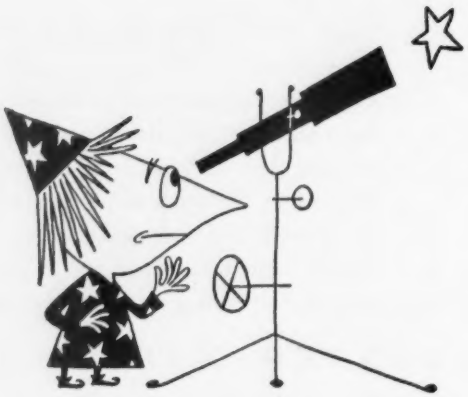

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ENGINEERING DEPARTMENT MODERNIZATION

BY

WILLIAM S. CAMPBELL [1]

After reading the address entitled "Industry's Responsibility in the Shortage of Engineers" which appeared in the October 1953 issue of this JOURNAL,[2] I was prompted to write the following article. Those who are vitally interested in engineering department management can readily share the concern so well set forth.

There is, however, another aspect of the problem which seems to me to be a very important and practical one. That is the adverse attitude of the young engineering graduate toward work on the drawing board.

During a visit to an outstanding engineering university for the purpose of interviewing forthcoming graduates for employment with a well known engineering company, I was disturbed by a comment made by the Placement Director to the effect that these young engineers have been trained for executive positions and not for work on the drawing board. However, several men joined with the company and although I am now retired, I understand most of them are still there and are doing well.

Many young engineers may side-step the drawing board but engineering companies cannot. Design drawings and detail drawings for manufacturing purposes must be made. Drawing and mathematics constitute the universal language of the engineer and unless the young graduate gains experience through the proper preparation of drawings, he cannot fully speak the language of his profession.

The products of the company to which I have referred are such that a period of from five to ten years in the engineering department is deemed desirable training to qualify one to move on to sales engineering or plant management.

[1] Chief Engineer (Retired), Materials Handling Equipment Division, Link Belt Company, Philadelphia, Pa.

[2] HORACE P. LIVERSIDGE, "Industry's Responsibility in the Shortage of Engineers," JOUR. FRANKLIN INST., Vol. 256, p. 305 (1953).

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

What can management do to encourage more men to enter the engineering profession? What can be done to remove the stigma of the drawing board? Many engineering departments are provided with obsolete and sometimes unsightly equipment. To make matters worse, engineers as a group are not very good housekeepers. Walk from the engineering department to the adjoining departments such as sales, accounting, or purchasing, and you will, most likely, find attractive offices equipped with modern desks, chairs, files, etc.

Industry can, no doubt, do several things to encourage young men to enter and stay in the engineering profession. One thing could be to transform their engineering departments into more desirable places in which to work. Then, with more insistence on good housekeeping, the

FIG. 1. Front view of new type drafting table. Drawing surface of table measures 30 × 42 in.; height above floor is 31 in.

personnel will soon respond. No blame can be laid upon industry or engineers for the unsightly appearance of engineering departments in general. What we are faced with here is a tradition and it is always difficult to break with tradition.

The time may be overdue when the break must be made. This was the thought of the company I have already referred to. It has completely modernized and refurnished several of its engineering departments into pleasing places to look upon and attractive places in which to work.

Engineers have spent years in designing up-to-date and efficient equipment for others, but they have not thought sufficiently about modernizing the equipment they, themselves, use. They are so accus-

tomed to the conventional high drawing table and equally high stool that it is now difficult for them to conceive that, in most cases, they could do their work just as well on a drawing surface of office desk height while sitting in a suitable chair that would permit them to rest their feet on the floor. Some readers may think this is impossible, but it has been done successfully in one of the engineering departments of the company to which I have referred.

A desk-high drawing desk is shown in Figs. 1 and 2. Several of these were constructed for trial purposes and their advantages were so apparent that a complete installation was made as shown in Fig. 3.

Experience has shown that drawings up to 22 in. × 34 in. can be made efficiently and with ease on this drawing desk. This size drawing is the largest drawing made in the department. It is a standard A.S.A.

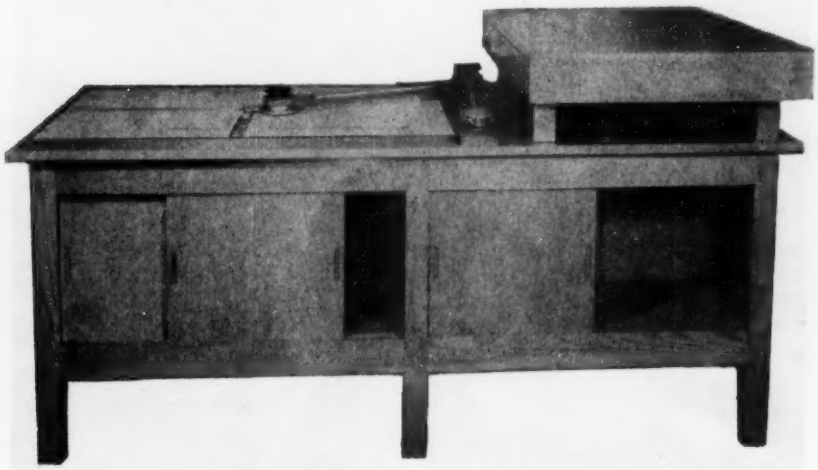

FIG. 2. Rear view of new type drafting table.

size and was adopted because prints made from the drawings can be folded to standard letter size $8\frac{1}{2}$ in. × 11 in. Other sizes commonly made are 17 in. × 22 in., 11 in. × 17 in., and $8\frac{1}{2}$ in. × 11 in., or one half, one quarter, and one eighth the size of the largest.

Many industries do not make larger drawings than 22 in. × 34 in. The smaller sizes are finding increasing favor because of their adaptability for drawings of standardized and classified parts.

The drawing surface of the desk is a cream-colored linoleum pad 30 in. × 42 in. Adhesive drafting tape is used to fasten the drawing paper to the linoleum pad. Thumb tacks or staples are not used. The engineer has for his use the drawers, cabinet space, and the slide in the front of his desk and in addition, he can utilize the cabinet space in the back of the desk behind him.

Other engineering departments may not have the need for so much storage space but, of course, this can be reduced as desired without destroying the advantages of the desk. The long drawer on the top at the left hand end can be replaced by a board of equal area. This would preserve the space for reference drawings as well as the space in which the arms of the drawing machine can operate.

The stigma of the drawing board must be removed. Possibly the most effective places to break with tradition would be in schools of engineering, where drawing sizes are small and where the low drawing desk would be an acceptable innovation. The objection of the young engineer to the conventional drawing table may be well founded. To

FIG. 3. Engineering Department using the new type drafting tables.

sit, uncomfortable, on a high stool with his feet resting on a ring attached to the stool is not too easy to accept while the adjacent fellow worker, possibly an office boy, sits comfortably at a standard office desk.

It must be remembered that the work at the drafting table is not all drafting. Many hours are spent in writing specifications, material lists, and calculation, all of which could more suitably be done at desk height.

Prior to half a century ago, the bookkeeper worked at a high desk and sat on a correspondingly high stool. This past practice has long been swept into the realms of history. Is the present day engineer in the grip of tradition as was the bookkeeper of fifty years ago? Could it be that the low drawing desk is long overdue?

THE STORAGE OF ENERGY IN SOME ACTIVATED ALKALI HALIDE PHOSPHORS *

BY

H. O. ALBRECHT [1] AND C. E. MANDEVILLE [1]

ABSTRACT

The phosphorescent emission of thallium activated NaCl and KCl has been investigated at 25° C., using as primary excitants alpha particles and ultraviolet light. The energy storage properties of X-ray irradiated NaCl-Tl, KCl-Tl, NaCl-Ag, and KCl-Ag have been compared, and the photostimulated ultraviolet emission of NaCl-Ag has been determined as a function of the silver concentration. Some dosimetric aspects of the data are discussed.

INTRODUCTION

The alkali halides are universally known as efficient phosphors for use in scintillation counting. A particular example is, of course, NaI-Tl. There are other properties of these materials which are useful though indirectly related to the fluorescent emission. Two such characteristics are:

(1) Phosphorescent afterglow, emission of light from the excited phosphor after cessation of irradiation.
(2) Energy storage in the irradiated phosphor.

The phosphorescence of item (1) above results from the escape of electrons from shallow traps. The kinetic energy of escape is supplied by the temperature of the surroundings in which the phosphor finds itself. Thus the phosphorescent emission at liquid nitrogen temperature is far less than that encountered at room temperature (25° C.). If the trap depths are sufficiently great, thermal agitation at room temperature may be insufficient to dislodge the trapped electrons which will remain "stored" in excited states in the solid. They will escape, however, if the phosphor is heated (thermostimulation) or irradiated by near ultraviolet, visible, or red light (photostimulation).

The primary excitant may take any one of several forms: X-rays, beta rays, gamma rays, alpha particles, or ultraviolet light. Irradiation by the more energetic excitants removes the electrons from the filled band, placing them in the conduction band of the phosphor. These electrons may be subsequently trapped in imperfections, forming, for example, *F*-centers, *F'*-centers, etc. The ultraviolet light, however, is often not energetic enough to bring about this condition through

* Assisted by the joint program of the Office of Naval Research and the Atomic Energy Commission.

[1] Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa.

absorption of a single photon. It may, however, excite to metastable states bound electrons in the heavy activator ion.

Whether metastable state or imperfection trap, the potential barrier for escape may be so great that thermal liberation at room temperature is very rare. The idea immediately presents itself that the alkali halides which exhibit storage could serve as dosimeters for nuclear radiation. After receipt of an initial burst of nuclear radiation, the irradiated phosphor can be interrogated days, weeks, or months afterwards to ascertain the previously received dosage.

Various aspects of the crystal dosimeter problem have been discussed in several previous publications by other authors. The use of alkali halides as the dosimetric material has already been considered in some detail. In particular, Kallmann *et al.* (1)[2] have reported on many characteristics of the silver activated alkali halides, and the measurement of dosages by them has also been similarly described by Friedman and Glover (2). The silver activated alkali halides have been reported to have particularly good storage properties, though storage is not limited to them alone.

Many of the alkali halide phosphors are double banded; that is, emission is to be found in both ultraviolet and visible bands. Under certain circumstances, the two bands can be studied separately in photomultiplier tubes with the aid of properly chosen filters. However, a more convenient method of separating the two bands is to use a photosensitive Geiger counter for ultraviolet detection and a photomultiplier tube such as the RCA-5819 for measurement of the visible emission. The RCA-1P28 is also of considerable value, because it responds to both visible and ultraviolet. When enclosed in a cylinder of soft glass, the 1P28 will, of course, respond to the visible radiation alone. The Geiger counter is of particular value in measurement of the photostimulated release of the "stored" ultraviolet, because it does not respond to the stimulating radiation. Measurement of the photostimulated emission can proceed simultaneously with stimulation. This advantageous feature of the photosensitive Geiger counter has been previously emphasized (2, 3, 4).

It should be remarked that the present investigation as well as those described in references 1, 2 and 3 are concerned with fluorescence, phosphorescence, and energy storage as related to irradiation by nuclear particles. Similar and closely related phenomena are to be associated with irradiation by ultraviolet. Silver and thallium activated alkali halide phosphors were intensively studied by Kato (4) and more recently by Etzel, Schulman, Ginther and Claffy (5). Many references to earlier papers in which ultraviolet was employed as the primary excitant can be found in the accounts of these latter investigations. In particular, it should be said that Kato used a photosensitive Geiger counter throughout the course of his work and pointed out specifically in the fourth

[2] The boldface numbers in parentheses refer to the references appended to this paper.

paper of his series the advantages of the spectral response of the Geiger counter in observing the photostimulated emission of ultraviolet while the stimulating near ultraviolet light is on.

All of the measurements described below, whether relating to spontaneous phosphorescence or long-period energy storage, were carried out at room temperature (25° C.). The exciting ultraviolet was always of wave length greater than 1800 Å, because the hydrogen arc was equipped with a quartz window.

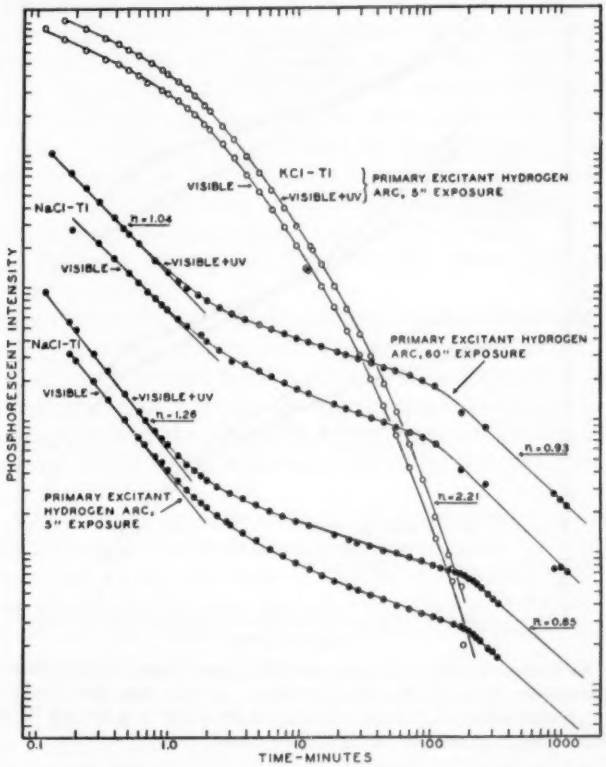

FIG. 1. Decay of NaCl + 0.1 per cent TlCl and of KCl + 0.1 per cent TlCl after excitation by uv. Excitation and decay at room temperature (25° C.). The decay of the visible band is detected in an RCA-1P28 phototube with soft glass filter and that of the ultraviolet band and the visible together in a 1P28 without filter.

THE THALLIUM ACTIVATED ALKALI HALIDES

A logical starting point for this discussion is the behavior of the thallium activated alkali halides, because they have already received so much attention (**6, 7, 8**). In particular, the phosphorescence of KCl-Tl at room temperature is considered to be dominated by a phosphorescent emission related to a trap depth of 0.67 ev and to excited states

of Tl^+ in close proximity to other Tl ions, probably Tl^+ ions adjacent in the crystal lattice (9). Since the excited electron never leaves the thallous ion of which it is a part, the decay is exponential in character, persisting above background for several hundred minutes. This type of behavior is exceptional to the usual in that exponential decay laws are commonly associated with half-periods of a few microseconds or less and seldom with a decay time of hours. The results of ultraviolet

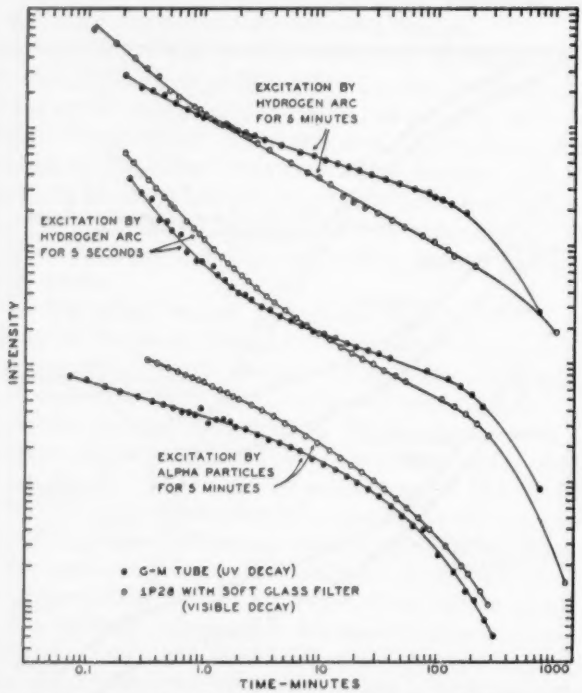

FIG. 2. Excitation of NaCl + 0.1 per cent TlCl (percentage TlCl concentration given by weight) by ultraviolet light and by alpha particles. In this case, the ultraviolet alone was detected in a photosensitive G-M tube, and the visible alone in a filtered RCA-1P28. The differing modes of decay of the two bands is to be noted.

irradiation of KCl-Tl and NaCl-Tl (thallium concentration 0.1 per cent by weight in either specimen) are shown in Fig. 1.[3] The decay law is entirely different in the two cases. The curves for NaCl-Tl could be considered a combination of exponentials and power laws. The curves for KCl-Tl are more nearly exponential on the log I–log t plot than are similar curves of many other phosphors studied at this laboratory. *However, the curve shape is not that of a single pure exponential plot.*

[3] Most of the phosphors employed to obtain the data of this paper were polycrystalline masses produced by fusion in a platinum crucible and subsequent rapid cooling on a glass plate. The characteristics of these phosphors were found to be much the same as those of single crystals. This has also been the experience of others (see reference 8).

[By way of explanation it should be said that to obtain the curves of Fig. 1, the irradiated phosphorescing material was placed between two 1P28 phototubes, one covered by a cylinder of soft glass. In this way the decay of the visible band alone could be measured in one tube and that of the visible and ultraviolet together in the other.] For the KCl-Tl phosphor, the decay law appears to be essentially the same in both spectral regions. The time of irradiation by ultraviolet from a hydrogen arc is given on the figure for each set of two decay curves. The slopes of decay on the log I–log t plot are indicated at various points. For example, at $t \sim 100$ min., the decay of KCl-Tl has assumed a slope of $n = 2.21$. It is to be particularly noted that on the time interval 2 min. $< t <$ 200 min., the slope on the log I–log t plot of the decay curve of the visible band of NaCl-Tl is greater than is the same quantity associated with the decay of the ultraviolet band; that is, the slope is "steeper." This effect is studied further in Fig. 2, where additional curves concerning NaCl-Tl are presented. To obtain these curves, the 1P28 tube detecting uv and visible light together was replaced by a G-M tube responding only to the ultraviolet. The relative "flatness" of the pure ultraviolet decay on the above mentioned time interval is thus enhanced.[4]

In the case of all of the ultraviolet excited curves concerning NaCl-Tl in Figs. 1 and 2, a rapid decay is in evidence at times below one minute. This mode of decay does not appear, however, in the case of the alpha particle excited curve of Fig. 2. From the various curves of Figs. 1 and 2 it is apparent that the intensity of the short-period decay relative to the remainder of the decay curve is reduced as the excitation time is increased.

In Fig. 3, the alpha-particle-excited phosphorescence of KCl-Tl is compared with the ultraviolet-excited phosphorescence. From the very similar appearance of the two sets of curves, it can be concluded that the phosphorescence can be very probably attributed to the same mechanism. Moreover, the visible and ultraviolet decay laws appear to be

[4] In an earlier publication (see reference 11) the writers described experiments which were designed to show that the ultraviolet band of NaCl-Ag follows a law of decay which differs from that of the visible band. The published measurements were performed with the use of a photosensitive G-M tube as the ultraviolet detector and a photomultiplier as the detector of visible light. In the course of the measurements (reference 11), the question arose whether the differing decay laws were real or were purely instrumental in nature. It was thought that perhaps the photosensitivity of the G-M tube could vary with counting rate. To test this point, after the paper of reference 11 was submitted for publication, the G-M tube was replaced by an RCA-1P28 phototube. The decaying NaCl-Ag was placed between the two RCA-1P28's, one filtered by a soft glass filter, the other unfiltered. The decay curve of ultraviolet and visible together was found to be "steeper" than the curve of visible light alone. When the soft glass jacket was transferred from one phototube to the other, the differing decay laws were again encountered, attesting the reproducibility and general validity of the phototube measurement and confirming the earlier measurement with the photosensitive G-M tube.

very nearly the same. Again, it should be pointed out that the ultraviolet measurements of Fig. 3 were obtained with a photosensitive G-M tube, whereas using the RCA-1P28, ultraviolet and visible bands were detected together in the unfiltered curves of Fig. 1.[5]

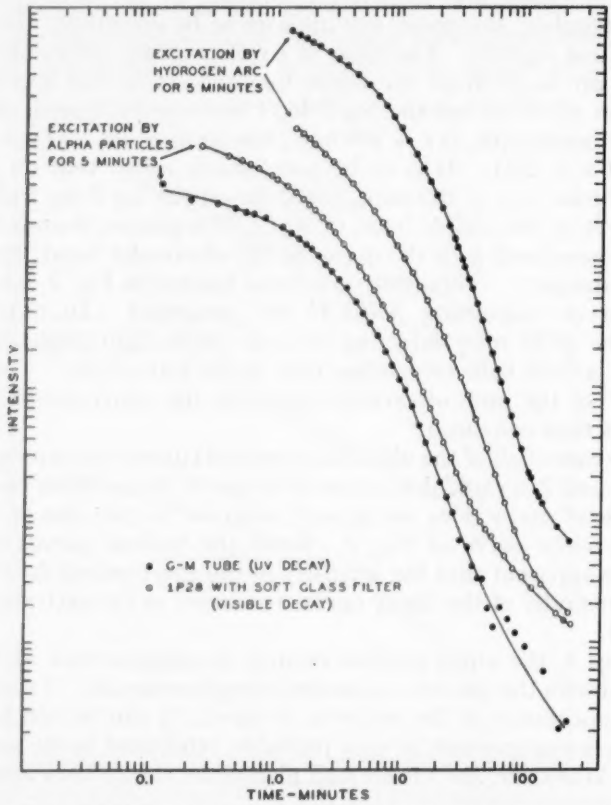

FIG. 3. Decay of KCl + 0.1 per cent TlCl after excitation by alpha particles and ultraviolet light. The decay of visible and ultraviolet emission has been studied separately by a proper choice of detectors.

A new crystalline melt was used in obtaining the data of each of Figs. 1, 2 and 3. For example, to obtain the measurements of Fig. 2, a specimen of NaCl-Tl was twice excited by ultraviolet and finally by alpha particles. The irradiated material was, of course, completely de-excited by photostimulation between the successively obtained sets of curves.

[5] The two curves of each set of curves of Figs. 1, 2, and 3 were always recorded simultaneously by placing the phosphorescing material between the two detectors. For example, when the detectors were RCA-1P28 with soft glass filter and photosensitive G-M tube, the visible band was recorded in the 1P28 and the ultraviolet band was recorded simultaneously with it in the G-M tube.

Irradiated samples of KCl-Tl and NaCl-Tl were examined at a time many hours after initial excitation to ascertain the extent of energy storage or "light storage" in the crystalline material. In this instance samples of KCl-Tl and NaCl-Tl were irradiated for about one hour with X-rays to receive a total dosage of ~20r. The excited samples were

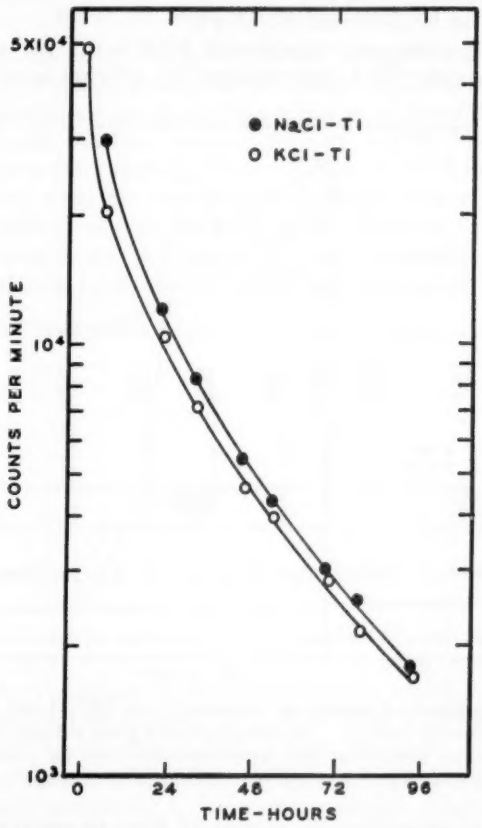

FIG. 4. Photostimulated emission of ultraviolet light from NaCl-Tl and KCl-Tl following excitation by a dosage of 20r of X-rays of maximum energy 25 Kev. The emission is observed simultaneously with photostimulation during periods of 6 sec. spaced at intervals of several hours.

then placed in a dark cavity at a distance of 7 cm. from a one-watt tungsten lamp. The photostimulated emission of ultraviolet was recorded in a Geiger counter during a period of 6 seconds while the stimulating light was on. This observation of photostimulated ultraviolet, emitted in six-second bursts, was repeated at intervals of several hours over a time of about 96 hours to obtain the curves of Fig. 4. From the

curves it is clear that after three days a counting rate of several thousand counts per minute is obtainable from either phosphor with the use of the relatively weak source of stimulating light. Measurements have shown that the number of stored electrons are appreciably reduced by the six-second period of photostimulation so that the curves of Fig. 4 may be taken as only an approximate indication of the rate of decay of the stored light in KCl-Tl and NaCl-Tl.

In another experiment, samples of KCl + 0.1 per cent TlCl and NaCl + 0.1 per cent TlCl (percentages by weight) were irradiated for

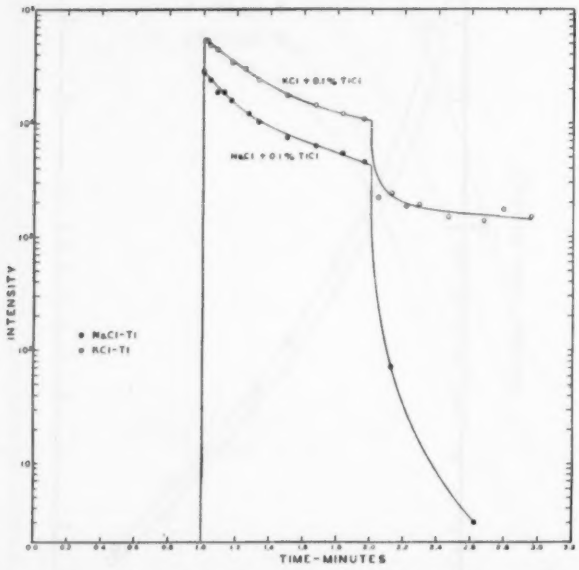

FIG. 5. Photostimulated emission of ultraviolet from KCl-Tl and NaCl-Tl (TlCl concentration 0.1 per cent by weight). The phosphors were given a dosage of 20r of X-rays and stored for 24 hr., after which time they were photostimulated for a period of one minute. KCl-Tl exhibits a post-stimulation afterglow of considerable intensity.

1 hr. by X-rays of maximum energy 25 Kev to receive a total dosage of ~20r. The irradiated materials were then stored in darkness for twenty-four hours. At the end of this time the samples were removed from storage, placed in a dark cavity, and irradiated with light of wavelength greater than 3600 Å from the tungsten lamp. The photostimulated emission of ultraviolet was recorded in a photosensitive Geiger counter as shown in Fig. 5. The ultraviolet emission of NaCl-Tl is seen to rise sharply, remain high while the stimulating light is on and drop to essentially nothing when the stimulating light is extinguished at the end of one minute. In contrast with this behavior, KCl-Tl exhibits after one minute of photostimulation, a very considerable post-

stimulation afterglow. The situation is just the reverse of that encountered in the case of the Ag-activated alkali halides (3); then, the post-stimulation afterglow is associated with NaCl-Ag, and the ultraviolet emission of KCl-Ag drops sharply with cessation of irradiation by the stimulating light (3).

THE SILVER ACTIVATED ALKALI HALIDES

Copious light emission and good storage properties are to be found among the Ag-activated alkali halides. When KCl-Ag is irradiated by ultraviolet or by nuclear particles the spectrum of luminescence is composed of two bands centered respectively at ~2800 Å and ~4350 Å. An emission spectrum resulting from alphas on KCl-Ag is shown in Fig. 6 as obtained with the use of a small Hilger quartz spectrograph.

A polycrystalline melt of KCl-Ag (AgCl content 0.1 per cent by weight), dimensions 1 cm. × 1 cm. × 0.5 cm., weighing one gram, was irradiated in darkness by 25 mc. of polonium alpha particles for 30 min.

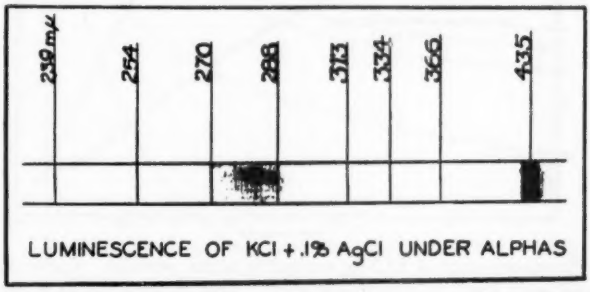

FIG. 6. Luminescent emission of KCl-Ag irradiated by alpha particles.

Immediately after cessation of bombardment by alphas, the sample of KCl-Ag was irradiated by filtered light ($\lambda > 3600$ Å) from the tungsten lamp having a total power dissipation of only one watt. The lamp was located seven centimeters from the KCl-Ag which was maintained at a constant temperature of 25° C. by water cooling. The decay of KCl-Ag under continuous photostimulation is shown in Curve *A* of Fig. 7. The detector was a photosensitive Geiger counter. In the early stages of the photostimulated decay, a considerable dead-time loss was encountered because of the high initial counting rate. The actual counting rate is roughly indicated by the broken line. From Curve *A*, it is clear that the crystal was exhausted of stored energy in about one thousand minutes of photostimulation by the one watt tungsten lamp. After complete de-excitation, the same crystal was again irradiated by polonium alpha particles, and the normal unstimulated phosphorescence was measured as a function of the time, as shown in Fig. 7, Curve *B*. It was estimated that 1.36×10^6 counts lay under Curve *A* and that

3.6×10^4 counts are under Curve *B*, only 2.6 per cent of Curve *A*. When the crystal of Curve *B* was exhausted by photostimulation commencing at 1500 minutes, an additional 3×10^5 counts were observed. According to Curve *A*, $\sim 10^6$ counts should have been recorded. The

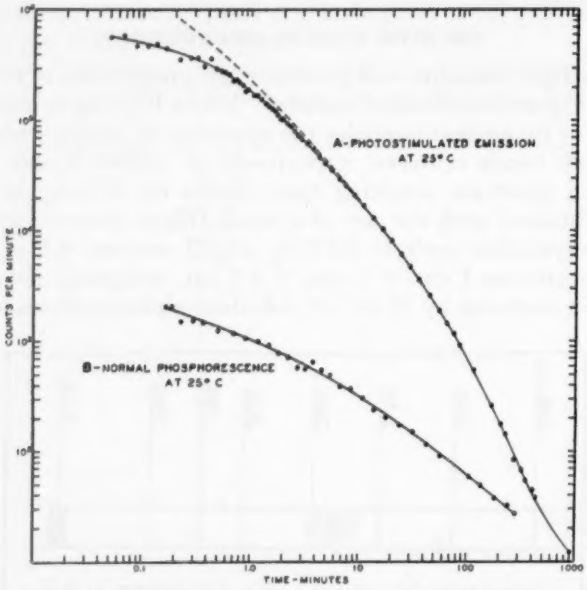

FIG. 7. Phosphorescence of KCl-Ag excited by alpha particles. Curve *A*, decay of the ultraviolet band under constant photostimulation; Curve *B*, spontaneous phosphorescence at room temperature.

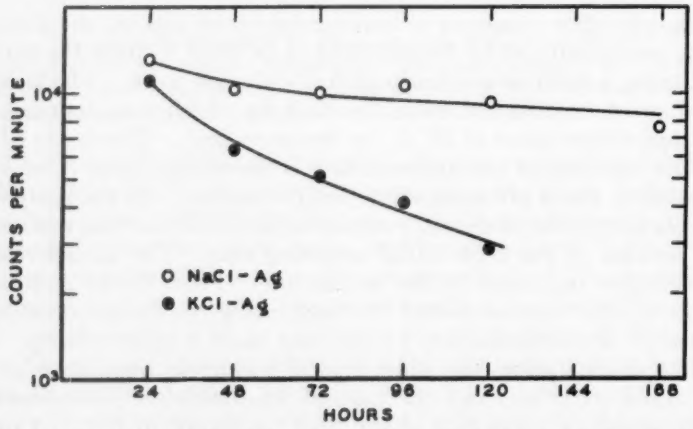

FIG. 8. Photostimulated emission of ultraviolet. Each phosphor had received a dosage of three roentgens at time zero.

result indicates that $\sim 7 \times 10^5$ counts or 70 per cent of the stored energy was lost through thermal processes in about 1500 minutes. In the course of the 30-min. bombardment by alphas, which preceded both Curve *A* and Curve *B* of Fig. 6, 2.06×10^6 counts were recorded in the G-M tube in the form of short lived fluorescence and unstimulated phosphorescence. The excitation, photostimulation, and counting were carried out with adequate exclusion of extraneous light.

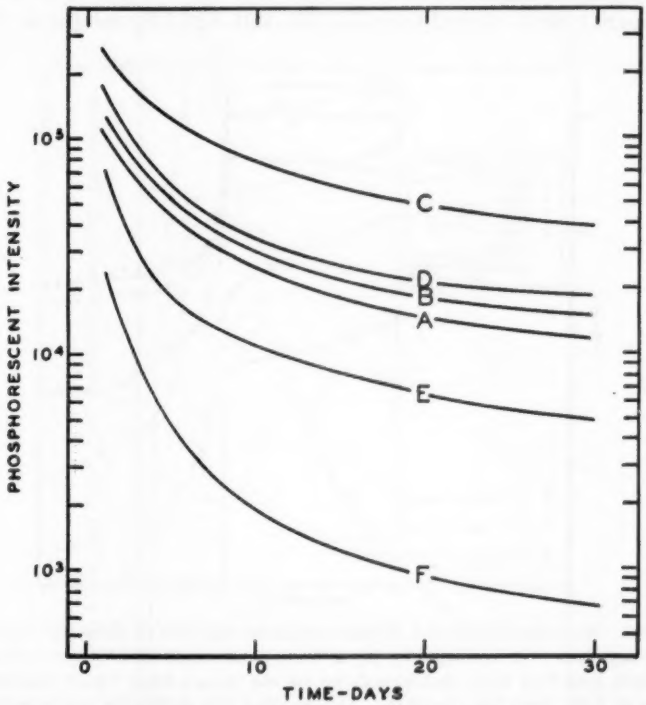

FIG. 9. Photostimulated emission of ultraviolet band of NaCl-Ag. Samples of identical volume were each given a dosage of 20r of X-rays. The emission was observed during a period of 6 sec. of photostimulation at intervals of 24 hr. The curves are lettered in order of decreasing silver concentration. See Fig. 10 for percentages of AgCl by weight.

A comparison of the storage properties of KCl + 0.1 per cent AgCl and NaCl + 0.1 per cent AgCl is shown in Fig. 8. The respective polycrystalline masses (equivalent in weight) of either phosphor had received a dosage of 3r during an excitation time of 10 min. The "stored light" was subsequently released, in 6-sec. bursts by photostimulation at intervals of 24 hr. The observation of photostimulated ultraviolet was made during the 6-sec. stimulation period. The counting rate thus obtained and plotted in Fig. 8 may be taken as a rough measure of the

amount of stored energy remaining in the crystal after an elapse of time specified by the axis of abscissas of the curves. From these two curves, it is clear that better storage properties are exhibited by NaCl-Ag than by KCl-Ag. There is, however, the post-stimulation afterglow (3) of NaCl-Ag which from the viewpoint of practical application could present a problem. This matter will be discussed later.

To ascertain what silver concentration leads to the best "light storage," polycrystalline melts of NaCl-Ag were prepared in which the silver concentration varied from 0.5 per cent AgCl by weight to 8×10^{-4}

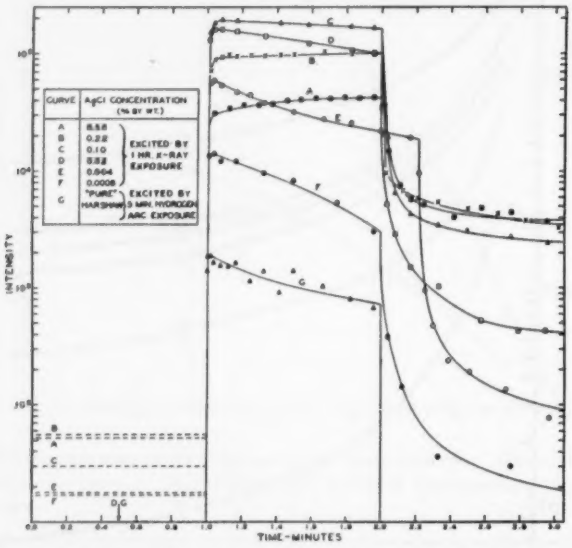

FIG. 10. Semi-logarithmic plot of photostimulated emission of ultraviolet band of NaCl-Ag. The materials were irradiated by 20r of X-rays and stored for 24 hr. in darkness, at the end of which time they were photostimulated for one minute by a 1-watt tungsten lamp at a distance of 7 cm. from the phosphor. The counting rate during the one minute of photostimulation and one minute thereafter is recorded. The highest counting rate is obtained for a silver chloride concentration of 0.10 per cent by weight.

per cent AgCl by weight. Identical quantities of each phosphor were irradiated for one hour to receive ~20r of X-rays.[6] They were stored in darkness and photostimulated by the one watt tungsten lamp for 6 sec. daily. The counting rate observed during the 6-sec. stimulation period is plotted in Fig. 9. The measurements were continued over a period of thirty days. From the curves of Fig. 9, it is clear that a silver concentration corresponding to addition of about 0.1 per cent AgCl by weight yields the maximum light emission during photostimulation. Here again, it should be made clear that as in the case of Fig. 4, the curves of Fig. 9 do not represent the actual decay of the stored energy

[6] The volume of the irradiated phosphors was usually 1 cm. × 1 cm. × 0.5 cm.

in NaCl-Ag. The amount of stored energy is perturbed by the measurements themselves. Each 6-sec. burst of photostimulated light drawn from the phosphor reduces in not negligible amounts the total of remaining stored energy. One sample of NaCl + 0.1 per cent AgCl showed, after five 6-sec. periods of photostimulation, about 15 per cent less activity than an identical specimen, which had not been previously photostimulated at all. The appreciable reduction of the stored light

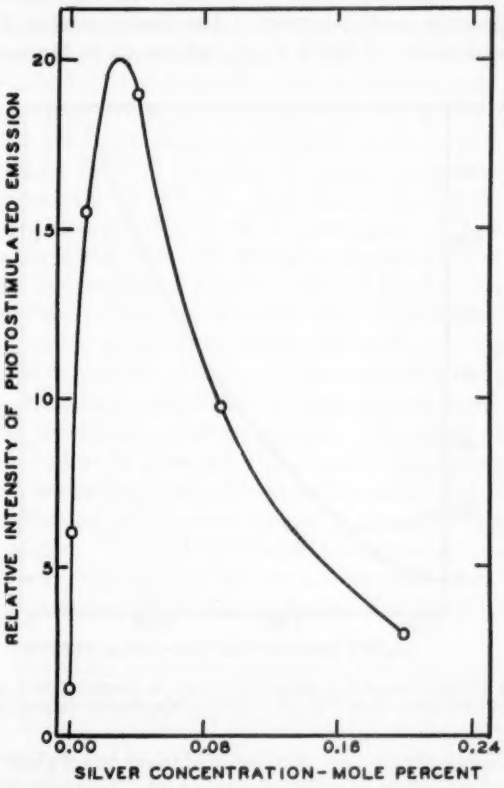

FIG. 11. Intensity of the photostimulated emission of the ultraviolet band of NaCl-Ag as a function of the concentration of AgCl in mole per cent.

by the brief periods of photostimulation could be, of course, eliminated by using a weaker tungsten bulb.

A second batch of polycrystalline samples of NaCl-Ag of differing silver concentration were similarly tested as shown in Fig. 10. In this case, the crystals were again irradiated to receive ~20r and stored in darkness. Twenty-four hours after excitation, each phosphor was photostimulated for about one minute. The ultraviolet emission was re-

corded during the one minute of photostimulation as well as afterwards to measure the post-stimulation afterglow. Here again, the initial rise in counting rate upon onset of photostimulation is greatest when the added AgCl is 0.1 per cent by weight. The various values of the initial counting rate ($t = 1$ min., time zero is taken as one minute before the photostimulating light is turned on) are plotted in Fig. 11. The maximum of ultraviolet emission between AgCl concentrations of 0.02 and 0.10 per cent by weight is clearly evident. In Fig. 11, the AgCl concentration is given in mole per cent. Another quantity obtainable from Fig. 10 is also plotted. This is C_2/C_1 where C_2 is the counting rate at

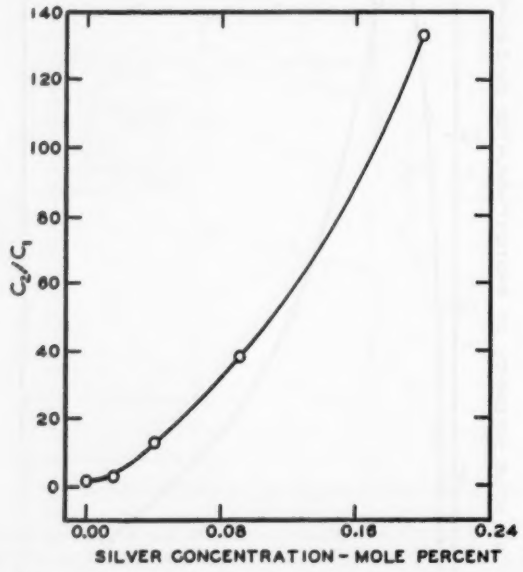

Fig. 12. Ratio of post-stimulation phosphorescence to co-stimulation phosphorescence. These data are taken from Fig. 10. C_2/C_1 is specifically defined in the text.

$t = 2$ min. arising from the post-stimulation afterglow and C_1 is the co-stimulation emission of ultraviolet at the instant of initial photostimulation at $t \sim 1$ min. The quantity C_2/C_1 is seen to increase with silver content. These data are shown in Fig. 12.

DISCUSSION OF RESULTS

From the standpoint of practical dosimetry, the data of the foregoing discussion should be of some value. It has been emphasized that the entire investigation including excitation and all measurements has been carried out at room temperature (25° C.), the situation which is met in practice. Many phosphors store energy at liquid nitrogen tem-

peratures, their filled traps emptying during the approach to room temperature. These materials would obviously have no practical value.

The curves of Fig. 9 show that certainly the storage is effective enough for accurate measurements. For example, the curve corresponding to an AgCl concentration of 0.10 per cent by weight, Curve *C*, indicates a counting rate of forty thousand counts per minute obtainable during photostimulation at the end of thirty days. Using a sufficiently weak stimulating light, counting rates of certainly several thousand counts per minute might be obtained without appreciably altering the amount of stored energy. Thus in the event of an error in making a first reading of the dosage, the phosphor could be "re-read." If successive readings are separated by only short time intervals, it is obvious that the post-stimulation afterglow such as that exhibited by NaCl-Ag (3) and by KCl-Tl (see Fig. 5), could yield misleading results. It has been previously shown (1), that NaCl-Ag can be re-used after having received a dosage of millions of roentgens. Thus, after having received a sizeable initial dose, the phosphor dosimeter can be de-excited and made ready for receipt of another dose of nuclear radiation. It is easy to visualize the collection of small light-tight capsules containing activated alkali halide dosimeters which had been previously located in an area subjected to atomic attack. These containers could be opened and read under standard conditions of photostimulation and detection to ascertain the previously received dosage. Knowing the time of the disaster, curves similar to those of Fig. 9 could be used to calculate the initial dose. It is evident that considerable work of a developmental nature must be carried out to standardize a phosphor dosimeter of the type described above. It would probably be necessary to develop single crystals of uniform response as well as a detection "kit," the components of which (counter, etc.) could be manufactured reproducibly in quantity.

Turning to the more technical aspects of the data, it might be said that it would be difficult to ascribe the bands of emission of the silver-activated alkali halides to definite excited states of the silver ion without detailed calculations. The ground state (10) of Ag^+ is $^1S_0$. The first group of excited states are *D* states which are sufficiently separated in energy from the ground state to account for the emission bands. However, direct transitions between *S* and *D* states are forbidden by parity considerations, if spectroscopic selection rules are to be followed. In an earlier paper (11), an attempt was made to explain the behavior of the phosphorescence of NaCl-Ag only by the presence of imperfection traps. This discussion cannot be regarded as a full interpretation, because the formation of metastable excited Ag^+ was not considered. The excited states of Tl^+ giving rise to the emission bands of KCl-Tl have been accurately identified (8).

It should perhaps also be remarked that the curve of Fig. 11 is very similar in general shape to the curves for luminescent efficiency versus activator concentration which have been calculated for various other phosphors (12). The specific case of NaCl-Ag has not yet been treated with a quantitative theoretical approach.

Note Added in Proof:

Since this paper was submitted for publication, an article by H. N. Bose and J. Sharma, *Proc. Phys. Soc. London*, Vol. 66B, p. 371 (1953), has appeared. They find the decay time of the emission spectrum of KCl-Tl to be a function of the wave length.

M. Furst and H. Kallmann have recently made additional studies of NaCl-Ag [*Phys. Rev.*, Vol. 91, p. 1356 (1953)]. They find that the photostimulated emission is an increasing function of activator concentration for amounts of AgCl as much as one percent by weight, whereas in the present paper the maximum occurred at about one-tenth percent. Consequently, the measurements of Fig. 9 were repeated with approximately the same result, differing as before from the observations of Furst and Kallmann.

Furst and Kallmann employed single crystals—fragments of commercially prepared ingots. Quantitative analyses of such fragments carried out in this laboratory have indicated a variation in activator concentration of a factor of two or three from the value stated by the manufacturer.

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A NEW ELECTRODYNAMICS

BY

PARRY MOON[1] AND DOMINA EBERLE SPENCER[2]

ABSTRACT

So successful have been Maxwell's equations that the electrodynamic formulations of Ampère, Weber, and Riemann have been almost forgotten. The purpose of this paper is to present a rejuvenation of the older theories, based entirely on the force between charged particles. The new formulation leads to the correct formulas for force, induced emf, and radiation. In fact, it may be regarded as an alternative to Maxwell's equations, with the advantages of Galilean relativity and a closer contact with reality (charges rather than fictitious flux lines).

1. INTRODUCTION

In the early part of the 19th century, Ampère (1),[3] Gauss (2), Weber (3), and Riemann (4) developed a theory of electrodynamics, free from the magnetic-field fiction and based on an extension of the familiar equation of Coulomb. At that time, the electron was unknown and ideas of metallic conduction were of the haziest kind; yet a true particle-theory was produced, both magnetic flux and the aether being ignored.

The attitude of these early investigators was surprisingly modern: their hard-headed phenomenological approach is much closer to the spirit of modern physics than is the mechanistic pictorialism of Faraday and Maxwell. It cannot be denied that the visualization of magnetic flux lines has been a genuine aid in electrical engineering. But there are advantages, both theoretical and practical, in the direct calculation of inter-particle forces in the pre-Maxwellian manner. It is interesting to see what can be done by a modernization of these older methods.

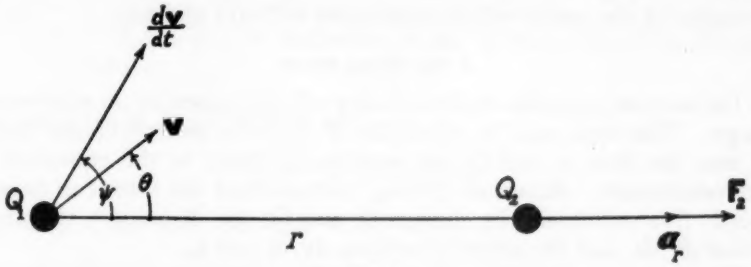

FIG. 1.

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[3] The boldface numbers in parentheses refer to the references appended to this paper.

Consider two charged particles (Fig. 1) separated by distance r. If there is no relative motion between the particles, Coulomb's equation applies. In the rationalized mks system,

$$\mathbf{F}_2 = \mathbf{a}_r \frac{Q_1 Q_2}{4\pi\epsilon r^2}. \tag{1}$$

This force is called the *Coulomb force.*

If Q_1 is moving at constant velocity $\mathbf{v}$ with respect to Q_2, the *Ampère force* resulting from this motion is

$$\mathbf{F}_2 = \mathbf{a}_r \frac{Q_1 Q_2}{4\pi\epsilon r^2}\left(\frac{v}{c}\right)^2[1 - \tfrac{3}{2}\cos^2\theta], \tag{2}$$

where θ is the angle between $\mathbf{v}$ and the unit vector $\mathbf{a}_r$. We have shown (5) that Eq. 2 is the only equation that is consistent with the Ampère experiments and with modern ideas of electronics.

The Ampère force is a substitute for the magnetic field. Equation 2 handles all problems dealing with forces on conductors carrying direct current. Equations 1 and 2 cover the questions that are ordinarily treated under electrostatics and magnetostatics. To complete the theory, however, we must introduce additional terms dealing with the *acceleration* of charge and with the *time-variation* of charge.

Consider all possible forces that can be exerted by charge Q_1 on charge Q_2 (Fig. 1). Evidently the possibilities may be classified as

(*a*) constant Q_1, no relative motion.
(*b*) constant Q_1, uniform relative velocity.
(*c*) constant Q_1, accelerated motion.
(*d*) Q_1 a function of time.

Condition (*a*) gives the Coulomb force, (*b*) the Ampère force. The remainder of the paper will be concerned with (*c*) and (*d*).

2. THE WEBER FORCE

The next step consists in introducing a force caused by an accelerated charge. This force may be called the *Weber force*, since Wilhelm Weber (3) was the first to include an acceleration term in the equations of electrodynamics. Reference to Fig. 1 shows that the force can depend on only five variables: the charges Q_1 and Q_2, the distance r, the acceleration $d\mathbf{v}/dt$, and the angle ψ between $d\mathbf{v}/dt$ and $\mathbf{a}_r$.

The requirement of linearity means that the force must be directly proportional to Q_1Q_2. The Weber force is directly proportional to acceleration because induced voltage is directly proportional to dI/dt or to $Q\,dv/dt$. Dimensional analysis then proves that the Weber force is inversely proportional to the first power of r and inversely proportional

to c^2. Without loss of generality, we can employ the constant of Eq. 1 and write for the Weber force,

$$\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon c^2 r} \frac{dv}{dt} \cdot \mathbf{F}(\psi), \tag{3}$$

where $\mathbf{F}(\psi)$ is an unknown vector function of the angle ψ.

Consider an element of conductor $d\mathbf{s}_1$ carrying a varying current (Fig. 2). The current is taken in the direction of the vector $d\mathbf{s}_1$, and

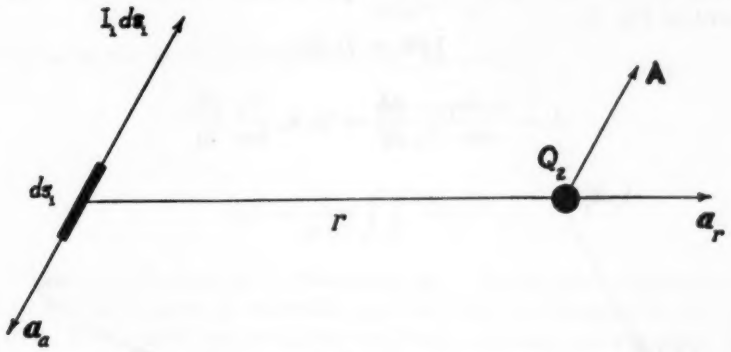

FIG. 2.

$$\begin{aligned} I_1 &= N_1 A_1 |Q_e| v_1, \\ dI_1/dt &= N_1 A_1 |Q_e| \, dv_1/dt, \end{aligned} \tag{4}$$

where

N_1 = number of free electrons per unit volume of conductor (m^{-3}),
A_1 = cross-sectional area of conductor (m^2),
$|Q_e|$ = magnitude of electronic charge (coulomb),
v_1 = drift velocity of electrons (m sec^{-1}) with respect to the conductor.

The electrons in ds_1 constitute a charge

$$Q_1 = -N_1 A_1 |Q_e| \, ds_1.$$

From Eq. 3, the force per unit charge on Q_2 is

$$\mathbf{F}_2/Q_2 = -\frac{N_1 A_1 |Q_e| \, ds_1}{4\pi\epsilon c^2 r} \frac{dv_1}{dt} \cdot \mathbf{F}(\psi)$$

and from Eq. 4,

$$\mathbf{F}_2/Q_2 = -\frac{ds_1}{4\pi\epsilon c^2 r} \frac{dI_1}{dt} \cdot \mathbf{F}(\psi). \tag{5}$$

Perhaps the simplest way of determining the unknown function $\mathbf{F}(\psi)$ is to compare Eq. 5 with the corresponding expression based on Maxwell's equations. According to Maxwell's theory,

$$\begin{cases} \mathbf{A} = \dfrac{1}{4\pi}\displaystyle\int \frac{\mathbf{J}}{r}\, d\mathcal{V}, \\ \mathbf{F}/Q = -\left[\nabla\varphi + \mu\dfrac{\partial \mathbf{A}}{\partial t}\right], \end{cases}$$

where $\mathbf{A}$ is the vector potential and $\mathbf{J}$ is the current density. For the element of Fig. 2,

$$\mathbf{J}\, d\mathcal{V} = I_1\, d\mathbf{s}_1,$$

so

$$\mathbf{A} = \frac{I_1\, d\mathbf{s}_1}{4\pi r}, \quad \frac{\partial \mathbf{A}}{\partial t} = -\,\mathbf{a}_a \frac{ds_1}{4\pi r}\frac{dI_1}{dt},$$

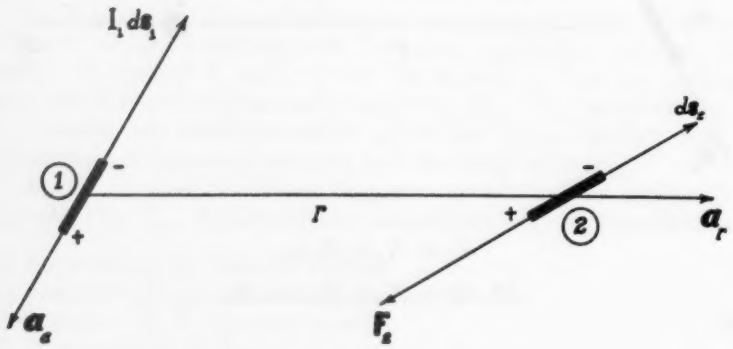

FIG. 3.

where $\mathbf{a}_a$ is a unit vector in the direction of the electron acceleration.

Since the element $d\mathbf{s}_1$ is uncharged, the scalar potential is zero and

$$\mathbf{F}_2/Q_2 = -\frac{1}{\epsilon c^2}\frac{\partial \mathbf{A}}{\partial t} = \mathbf{a}_a \frac{ds_1}{4\pi\epsilon c^2 r}\frac{dI_1}{dt}. \tag{6}$$

Comparison of Eqs. 5 and 6 shows that

$$\mathbf{F}(\psi) = -\,\mathbf{a}_a.$$

Thus the final equation for the Weber force for two charged particles is, from Eq. 3,

$$\mathbf{F}_2/Q_2 = -\,\mathbf{a}_a \frac{Q_1}{4\pi\epsilon c^2 r}\frac{dv}{dt}, \tag{7}$$

where v is the relative velocity of charge 1 with respect to charge 2.

3. INDUCED EMF

The difference in potential between the ends of an element $d\mathbf{s}_2$ (Fig. 3), induced by the varying current in $d\mathbf{s}_1$, is

$$dV_2 = -\mathbf{F}_2/Q_2 \cdot d\mathbf{s}_2,$$

where the emf is taken positive for a potential rise in the direction of $d\mathbf{s}_2$. From Eq. 6, the induced emf in $d\mathbf{s}_2$ is

$$d^2V_2 = -\frac{d\mathbf{s}_1 \cdot d\mathbf{s}_2}{4\pi\epsilon c^2 r}\frac{dI_1}{dt}. \tag{8}$$

Introducing the mutual inductance M, we obtain

$$d^2M = \frac{d\mathbf{s}_1 \cdot d\mathbf{s}_2}{4\pi\epsilon c^2 r}$$

or

$$M = \frac{1}{4\pi\epsilon c^2}\int\int \frac{d\mathbf{s}_1 \cdot d\mathbf{s}_2}{r}. \tag{9}$$

This is the equation of F. Neumann (6). Equation 9 applies equally well to self inductance L when $d\mathbf{s}_1$ and $d\mathbf{s}_2$ refer to elements in the same circuit. The result was obtained without reference to magnetic flux, and it is sometimes more convenient than the conventional method of flux linkages. Grover (7) says of Eq. 9:

> This is the most general expression for finding the mutual inductance. It leads quite simply to a *formal* expression for the mutual inductance even though for most cases it is not possible to perform the integrations [analytically]. . . . For inclined filaments . . . the Neumann formula has the advantage, and the formula for the mutual inductance of two straight filaments placed in any desired position has also been obtained by its use.

Thus the Weber force, caused by the acceleration of a charged particle, is given by Eq. 7. The emf induced in an element $d\mathbf{s}_2$ is given by Eq. 8. The mutual inductance between closed circuits of any form is expressed by Eq. 9, and the induced emf is

$$V_2 = -M\frac{dI_1}{dt}. \tag{10}$$

As an illustration, consider the mutual inductance between two coaxial circles (Fig. 4). Take a charge Q_2 at a fixed point P on the outer circle; and determine the force on this charge, caused by a varying current in the inner circle.

According to Eq. 6,

$$d\mathbf{F}_2/Q_2 = -\frac{d\mathbf{s}_1}{4\pi\epsilon c^2 r'}\frac{dI_1}{dt}, \tag{6a}$$

where $ds_1 = a\,d\zeta$, $r'^2 = a^2 + b^2 + z^2 - 2ab\cos\zeta$. The tangential component of force at P, caused by current in the complete loop of radius a, is

$$F_t/Q_2 = -\frac{a}{4\pi\epsilon c^2}\frac{dI_1}{dt}\int_0^{2\pi}\frac{\cos\zeta\,d\zeta}{r}$$

$$= -\frac{a}{2\pi\epsilon c^2}\frac{dI_1}{dt}\int_0^{\pi}\frac{\cos\zeta\,d\zeta}{[(a^2+b^2+z^2) - 2ab\cos\zeta]^{\frac{1}{2}}}.$$

Let $\zeta = \pi - 2\varphi$, $\cos\zeta = -\cos 2\varphi = -(1 - 2\sin^2\varphi)$, $d\zeta = -2\,d\varphi$. Then

$$F_t/Q_2 = \frac{a}{\pi\epsilon c^2}\frac{dI_1}{dt}\int_0^{\pi/2}\frac{(1-2\sin^2\varphi)\,d\varphi}{[(a+b)^2 + z^2 - 4\,ab\sin^2\varphi]^{\frac{1}{2}}}$$

$$= \frac{a}{\pi\epsilon c^2[(a+b)^2+z^2]^{\frac{1}{2}}}\frac{dI_1}{dt}\int_0^{\pi/2}\frac{(1-2\sin^2\varphi)\,d\varphi}{[1-k^2\sin^2\varphi]^{\frac{1}{2}}},$$

where

$$k^2 = 4ab/[(a+b)^2 + z^2]. \tag{11}$$

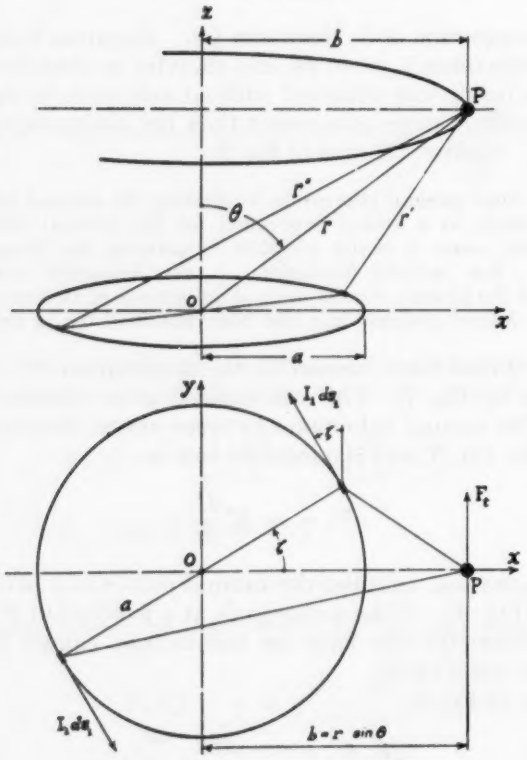

FIG. 4.

Integration leads to

$$F_t/Q_2 = -\frac{a}{\pi\epsilon c^2 k^2[(a+b)^2+z^2]^{\frac{1}{2}}}\frac{dI_1}{dt}[(2-k^2)F-2E], \quad (12)$$

where F and E are complete elliptic integrals of the first and second kinds. Equation 12 gives the force per unit charge on a particle at P, produced by a varying current in the loop of radius a.

The emf induced in a complete loop of radius b is

$$\begin{aligned} V &= \oint (F_t/Q_2)\, ds_2 = 2\pi b(F_t/Q_2) \\ &= -\frac{2ab}{\epsilon c^2 k^2[(a+b)^2+z^2]^{\frac{1}{2}}}\frac{dI}{dt}[(2-k^2)F-2E], \end{aligned} \quad (13)$$

so the mutual inductance of two concentric circles is

$$M = \frac{1}{2\epsilon c^2}[(a+b)^2+z^2]^{\frac{1}{2}}[(2-k^2)F-2E]. \quad (14)$$

This is exactly the equation obtained by Maxwell. It could have been found, of course, directly from Eq. 9, though the above derivation gives a better physical picture of what is taking place.

4. THE LOOP ANTENNA

The induced emf in a circular loop is considered in Section 3. The equations apply to relatively low frequencies so that the time lag caused by the finite velocity of propagation may be neglected. We now take the case where retardation must be included but where the current is still essentially in phase around the loop.

The unretarded Weber force per unit charge is

$$d\mathbf{F}/Q_2 = -\frac{d\mathbf{s}_1}{4\pi\epsilon c^2 r}\frac{dI(t)}{dt} \quad (6a)$$

and the retarded Weber force is therefore

$$d\mathbf{F}/Q_2 = -\frac{d\mathbf{s}_1}{4\pi\epsilon c^2 r}\frac{dI(t-r/c)}{dt}. \quad (6b)$$

Let $I(t) = \sqrt{2}I^* e^{i\omega t}$ and $F(t) = \sqrt{2}F^* e^{i\omega t}$, where I^* and F^* are rms values (generally complex numbers). Then

$$\frac{dI(t-r/c)}{dt} = i\omega\sqrt{2}I^* e^{i\omega t}e^{-\frac{i\omega r}{c}}.$$

Thus Eq. 6*b* becomes

$$d\mathbf{F}/Q_2 = -\frac{i\omega\sqrt{2}\, d\mathbf{s}_1 I^*}{4\pi\epsilon c^2 r} e^{i\omega t} e^{-\frac{i\omega r}{c}}$$

and the tangential force per unit charge (Fig. 4) is

$$dF_t^*/Q_2 = -\frac{i\omega\, ds_1 I^* \cos\zeta}{4\pi\epsilon c^2 r'} e^{-\frac{i\omega r'}{c}}. \tag{6c}$$

Consider two current elements situated diametrically opposite each other on a circle of radius a (Fig. 4). Distances from these elements to point P are r' and r''. The tangential force, produced at P by current in the pair of elements, is

$$dF_t^*/Q_2 = -\frac{i\omega a I^* \cos\zeta\, d\zeta}{4\pi\epsilon c^2}\left[\frac{e^{-\frac{i\omega r'}{c}}}{r'} - \frac{e^{-\frac{i\omega r''}{c}}}{r''}\right], \tag{15}$$

with

$$r'^2 = a^2 + r^2 - 2ar\sin\theta\cos\zeta,$$
$$r''^2 = a^2 + r^2 + 2ar\sin\theta\cos\zeta.$$

If $r \gg a$,

$$r' \cong r\left(1 - \frac{a}{r}\sin\theta\cos\zeta\right),$$
$$r'' \cong r\left(1 + \frac{a}{r}\sin\theta\cos\zeta\right),$$
$$\frac{1}{r'} \cong \frac{1}{r}\left(1 + \frac{a}{r}\sin\theta\cos\zeta\right),$$
$$\frac{1}{r''} \cong \frac{1}{r}\left(1 - \frac{a}{r}\sin\theta\cos\zeta\right).$$

Substitution in Eq. 15 gives

$$dF_t^*/Q_2 = -\frac{i\omega a I^* e^{-\frac{i\omega r}{c}} \cos\zeta\, d\zeta}{2\pi\epsilon c^2 r}\left[i\sin\left(\frac{\omega a}{c}\sin\theta\cos\zeta\right) + \frac{a}{r}\sin\theta\cos\zeta\cos\left(\frac{\omega a}{c}\sin\theta\cos\zeta\right)\right].$$

If $\omega a/c \ll 1$,

$$\sin\left(\frac{\omega a}{c}\sin\theta\cos\zeta\right) \cong \frac{\omega a}{c}\sin\theta\cos\zeta,$$
$$\cos\left(\frac{\omega a}{c}\sin\theta\cos\zeta\right) \cong 1.$$

Thus,

$$dF_t^*/Q_2 \cong -\frac{i\omega a^2[I^*]}{2\pi\epsilon c^2 r}\left[\frac{i\omega}{c}+\frac{1}{r}\right]\sin\theta\cos^2\zeta\,d\zeta, \tag{16}$$

where

$$[I^*] = I^* e^{-\frac{i\omega r}{c}}.$$

The tangential force per unit charge at P, caused by current in the complete loop of radius a, is

$$F_t^*/Q_2 = -\frac{i\omega a^2[I^*]\sin\theta}{\pi\epsilon c^2 r}\left[\frac{i\omega}{c}+\frac{1}{r}\right]\int_0^{\pi/2}\cos^2\zeta\,d\zeta$$

or

$$\mathbf{F}^*/Q_2 = \mathbf{a}_\psi\frac{\omega(\pi a^2)[I^*]\sin\theta}{4\pi\epsilon c^2 r}\left[\frac{\omega}{c}-\frac{i}{r}\right], \tag{17}$$

where $\mathbf{a}_\psi$ is a unit vector in the tangential direction. This equation agrees with the usual formula derived from Maxwell's equations. It shows that the force at P is directly proportional to the area of the loop and directly proportional to the sine of the angle from the loop axis. The first term of Eq. 17, which varies inversely as the first power of the distance, represents the ordinary radio wave. The second term varies inversely as the second power of r and thus becomes negligible at very large distances.

5. THE MAXWELL FORCE

The preceding sections have evaluated cases (*a*), (*b*), and (*c*) of Section 1. There remains the rather unusual case (*d*) of a charge that is not constant. For example, with a linear conductor in the stationary or quasi-stationary state, the conduction current entering an elementary length of conductor is equal to the current leaving it, in accordance with Kirchhoff's second law. But at sufficiently high frequency, this relation no longer holds: the charge on the element depends on both time and position. Because (*d*) is closely related to Maxwell's displacement current, the corresponding force will be called the *Maxwell force.*

Suppose that Q_1 of Fig. 1 varies sinusoidally with time:

$$Q_1(t) = \sqrt{2}Q^* e^{i\omega t}.$$

The corresponding retarded quantity is

$$Q_1(t - r/c) = \sqrt{2}Q^* e^{i\omega(t-r/c)}.$$

According to Maxwell's theory, this varying charge produces a force on Q_2:

$$\mathbf{F}^*/Q_2 = \mathbf{a}_r\frac{[Q^*]}{4\pi\epsilon}\left[\frac{1}{r^2}+\frac{i\omega}{cr}\right], \tag{18}$$

where

$$[Q^*] = \sqrt{2}Q^* e^{-i\omega r/c}, \quad F = \sqrt{2}F^* e^{i\omega t}.$$

This same result is obtained from the new theory if we assume for the Maxwell force,

$$\mathbf{F}/Q_2 = -\mathbf{a}_r \frac{1}{4\pi\epsilon} \frac{\partial}{\partial r}\left[\frac{1}{r} Q_1(t - r/c)\right]. \tag{19}$$

Then for the charged particles of Fig. 1, Eq. 19 gives

$$\mathbf{F}/Q_2 = -\mathbf{a}_r \sqrt{2} \frac{Q^*}{4\pi\epsilon}\left[-\frac{1}{r^2} - \frac{i\omega}{cr}\right] e^{i\omega(t-r/c)}$$

or

$$\mathbf{F}^*/Q_2 = \mathbf{a}_r \frac{[Q^*]}{4\pi\epsilon}\left[\frac{1}{r^2} + \frac{i\omega}{cr}\right]. \tag{18}$$

For the special case of $Q_1(t) = Q_1 = \text{const}$, Eq. 19 reduces to the Coulomb force,

$$\mathbf{F}/Q_2 = \mathbf{a}_r \frac{Q_1}{4\pi\epsilon r^2}. \tag{1}$$

Thus the Maxwell force includes the Coulomb force as a special case, and no separate term for the Coulomb force is needed.

Just as the Ampère force replaces the magnetic field, so the Maxwell force replaces the displacement current, though of course Eq. 19 was not formulated by Maxwell. In fact, none of the previous theories seems to include such a force, which may account for the rather unsatisfactory nature of the theories of Weber, Riemann, Ritz (8), and Warburton (9).

As an example, take an electric dipole (Fig. 5) with the charges varying sinusoidally with time:

$$Q = \sqrt{2}Q^* e^{i\omega t}.$$

For the upper charge, according to Eq. 19,

$$\mathbf{F}/Q_2 = \mathbf{a}_{r'} \frac{\sqrt{2}Q^*}{4\pi\epsilon}\left[\frac{1}{r'^2} + \frac{i\omega}{cr'}\right] e^{i\omega(t-r'/c)}.$$

But if $l \ll r$,

$$r' \cong r(1 - (l/2r)\cos\theta), \quad r'' \cong r(1 + (l/2r)\cos\theta),$$

$$\frac{1}{r'} \cong \frac{1}{r}(1 + (l/2r)\cos\theta), \quad \frac{1}{r''} \cong \frac{1}{r}(1 - (l/2r)\cos\theta).$$

Then

$$\mathbf{F}^*/Q_2 = \frac{I^* e^{-\frac{i\omega r}{c}} e^{\frac{i\omega l}{2c}\cos\theta}}{i\omega 4\pi\epsilon}\left\{\frac{1}{r^2}\left(1+\frac{l}{r}\cos\theta\right)+\frac{i\omega}{cr}\left(1+\frac{l}{2r}\cos\theta\right)\right\} \times\left[\mathbf{a}_r+\mathbf{a}_\theta\frac{l\sin\theta}{2r}\right]$$

where I is the current between the two spheres:

$$I=\sqrt{2}I^* e^{i\omega t},\quad Q=\frac{\sqrt{2}I^* e^{i\omega t}}{i\omega}.$$

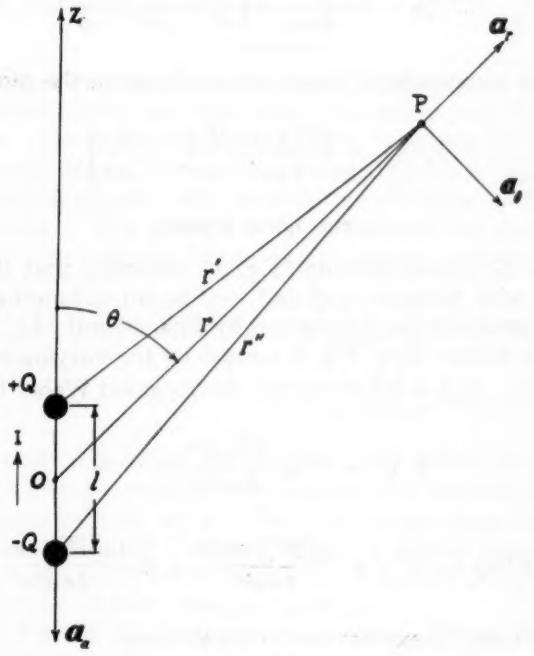

FIG. 5.

Similarly, for the lower charge,

$$\mathbf{F}^*/Q_2 = \frac{I^* e^{-\frac{i\omega r}{c}} e^{-\frac{i\omega l}{2c}\cos\theta}}{i\omega 4\pi\epsilon}\left\{\frac{1}{r^2}\left(1-\frac{l}{r}\cos\theta\right)+\frac{i\omega}{cr}\left(1-\frac{l}{2r}\cos\theta\right)\right\} \times\left[-\mathbf{a}_r+\mathbf{a}_\theta\frac{l\sin\theta}{2r}\right].$$

The sum of the two forces has a radial component,

$$F_r^*/Q_2 = \frac{I^* e^{-\frac{i\omega r}{c}}}{i\omega 4\pi\epsilon}\left\{\frac{1}{r^2}\left[2i\sin\left(\frac{\omega l}{2c}\cos\theta\right) + \frac{2l}{r}\cos\theta\cos\left(\frac{\omega l}{2c}\cos\theta\right)\right]\right.$$
$$\left. + \frac{i\omega}{cr}\left[2i\sin\left(\frac{\omega l}{2c}\cos\theta\right) + \frac{l}{r}\cos\theta\cos\left(\frac{\omega l}{2c}\cos\theta\right)\right]\right\}.$$

But

$$\sin\left(\frac{\omega l}{2c}\cos\theta\right) \cong \frac{\omega l}{2c}\cos\theta, \quad \cos\left(\frac{\omega l}{2c}\cos\theta\right) \cong 1,$$

so

$$F_r^*/Q_2 = \frac{[I^*]l\cos\theta}{4\pi\epsilon c}\left[\frac{2}{r^2} - \frac{i2c}{\omega r^3} + \frac{i\omega}{cr}\right]. \tag{20}$$

Similarly, the component of force perpendicular to the radius is

$$F_\theta^*/Q_2 = \frac{[I^*]l\sin\theta}{4\pi\epsilon c}\left[\frac{1}{r^2} - \frac{ic}{\omega r^3}\right]. \tag{21}$$

6. THE DIPOLE ANTENNA

Consider the dipole antenna (Fig. 5), assuming that the current in the vertical wire between $+Q$ and $-Q$ is not a function of z. The dipole itself produces the force given by Eqs. 20 and 21. To this must be added the Weber force, Eq. 6, caused by the varying current in the wire. For $I(t - r/c) = \sqrt{2}I^* e^{i\omega(t-r/c)}$, the retarded Weber force is

$$\mathbf{F}/Q_2 = -\mathbf{a}_z \frac{\sqrt{2}i\omega l I^*}{4\pi\epsilon c^2 r} e^{i\omega(t-r/c)}$$

or

$$(\mathbf{F}^*/Q_2)_W = -\mathbf{a}_r \frac{i\omega[I^*]l\cos\theta}{4\pi\epsilon c^2 r} + \mathbf{a}_\theta \frac{i\omega[I^*]l\sin\theta}{4\pi\epsilon c^2 r}. \tag{22}$$

Equations 20 and 21 express the Maxwell force:

$$(\mathbf{F}^*/Q_2)_M = \mathbf{a}_r \frac{[I^*]l\cos\theta}{4\pi\epsilon c}\left[\frac{2}{r^2} - \frac{i2c}{\omega r^3} + \frac{i\omega}{cr}\right]$$
$$+ \mathbf{a}_\theta \frac{[I^*]l\sin\theta}{4\pi\epsilon c}\left[\frac{1}{r^2} - \frac{ic}{\omega r^3}\right]. \tag{23}$$

The Ampère force is a double-frequency force because of the second power of the velocity, Eq. 2. Thus the total force at fundamental frequency is the sum of the Weber and the Maxwell forces, or

$$\mathbf{F}^*/Q_2 = \mathbf{a}_r \frac{[I^*]l \cos\theta}{2\pi\epsilon c}\left[\frac{1}{r^2} - \frac{ic}{\omega r^3}\right] + \mathbf{a}_\theta \frac{[I^*]l \sin\theta}{4\pi\epsilon c}\left[\frac{1}{r^2} - \frac{ic}{\omega r^3} + \frac{i\omega}{cr}\right]. \quad (24)$$

At very great distances, Eq. 24 reduces to

$$\mathbf{F}^*/Q_2 = \mathbf{a}_\theta \frac{i\omega[I^*]l \sin\theta}{4\pi\epsilon c^2 r}, \quad (24a)$$

showing that the force per unit charge at the receiver varies directly as the length of the antenna, directly as the frequency, and inversely as the distance. Equations 24 and 24*a* agree with the classical results obtained from Maxwell's equations.

7. SUMMARY

The paper has developed an electrodynamics that is in agreement with Galilean relativity and free from the aether and the magnetic-field concept. The fundamental equation expresses the force on a charged particle, caused by another charged particle (Fig. 1). The particles may be stationary with respect to each other, or they may be in relative motion. The charges may be constant or they may vary with time. In any case, the force per unit charge at point 2 is

$$\mathbf{F}/Q_2 = \mathbf{a}_r \frac{Q_1}{4\pi\epsilon r^2}\left(\frac{v}{c}\right)^2[1 - \tfrac{3}{2}\cos^2\theta] - \mathbf{a}_a \frac{Q_1}{4\pi\epsilon c^2 r}\frac{d}{dt}v(t - r/c) - \mathbf{a}_r \frac{1}{4\pi\epsilon}\frac{\partial}{\partial r}\left[\frac{1}{r}Q_1(t - r/c)\right]. \quad (25)$$

The first term represents the *Ampère force*, which is particularly important with direct currents. The second term represents the *Weber force*, which gives induced emf's. The third term represents the *Maxwell force*. It includes the *Coulomb force* as a special case where Q = const., but it is especially useful where the frequency is high enough so that Q is a function of both time and position.

Note that Eq. 25 is inherently relativistic, without the need of Lorentz contraction or Einstein pseudo-relativity. The only velocity is the *relative* velocity $\mathbf{v}$ of charge 1 with respect to charge 2; the only acceleration is the rate of change of relative velocity. Equation 25 may be used as the basis for all of electrodynamics. Thus questions dealing with aggregates of charge are handled by summation (or integration) of F/Q_2, and problems dealing with currents are treated by considering large numbers of electrons in motion. The only limitation of Eq. 25 seems to be that it applies only to ordinary velocities. It is probable that the functions of Eq. 25 will require slight modification if they are applied to electrons at extreme velocities.

The new formulation of electrodynamics is built on the foundations laid by Weber and by Ritz. It differs from all previous theories, however, particularly in the term for the Maxwell force. This formulation may be regarded as an alternative to Maxwell's equations. Maxwell based his work on the closed circuit, on the aether, and on the Faraday visualization of flux lines. The Maxwell equations give the correct answers to a great number of questions; but the application of these equations to open circuits is sometimes ambiguous and uncertain. Particularly in considering the magnetic field produced by electrons in motion, one may find the classical field concepts to be clumsy tools. The new formulation generally leads to exactly the same results as Maxwell's equations, but in some cases it may give a more direct approach and one that is free from ambiguity.

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ON THE FLOW BEHIND AN AXIALLY SYMMETRICAL ATTACHED CURVED SHOCK*

BY

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SUMMARY

The flow behind a curved shock attached to a body of revolution placed in a uniform stream whose direction is in the direction of the axis of the body is investigated by considering the perturbation of the stream function in the Taylor-Maccoll solution for a cone.

The flow patterns are divided into three groups: One is designated as the "weak shock" case in which the resultant velocity of the corresponding conical flow behind the shock is always supersonic; the second is designated as the "strong shock" case in which the resultant velocity of the basic conical flow behind the shock is always subsonic; and the third is the transonic case. Solutions for these three cases are obtained for arbitrary shape of the body of revolution.

In the transonic case, a special first order differential equation for the perturbed stream function valid in the neighborhood of Mach number equal to one is found. This equation is used to join the solutions for the supersonic and the subsonic flows so that a smooth transition from supersonic to subsonic flow is obtained.

I. INTRODUCTION

The essential feature of flow behind a curved shock is that the flow is rotational with non-uniform entropy. One of the difficulties in solving the problem of flow behind a curved shock is that the distribution of entropy in the flow is in general not known *a priori* and depends on the solution of the problem. Another difficulty is the non-linearity of the differential equation that governs the flow of compressible fluid. A general theory[2] has been previously developed by the author to solve this problem in which the differential equation of the stream function of rotational flow is linearized by the method of perturbation over certain irrotational flow; the linearized equation is solved by the method of iteration.

In this paper we apply this method to investigate the flow behind a curved shock attached to a body of revolution in a uniform supersonic stream. The flow is considered to be steady, inviscid and adiabatic. Since the flow in front of the curved shock is assumed to be uniform, that is, isentropic and irrotational, the whole flow field is isoenergetic.

We consider a uniform supersonic flow of ideal gas passing over a body of revolution in the direction of the axis of revolution of the body. If the leading edge angle of the body is smaller than the critical angle, a curved shock front is attached to the vertex of the body. Behind the curved shock, the flow is rotational with non-uniform entropy. From

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[2] S. I. Pai, "On the Flow Behind Curved Shock," Cornell Aeronautical Laboratory Report, 1949 (unpublished); also, *Jour. Aero. Sci.*, Vol. 19, No. 11, pp. 734–742 (1952).

the shock polar it is well known that for a given angle of deflection of the flow, there are two possible configurations of shock angle, the strong shock and the weak shock. Behind the weak shock, the flow is usually supersonic, while behind the strong shock, the flow is always subsonic. These two cases should be treated separately since they involve a change of type, from hyperbolic to elliptic, in the fundamental differential equation of the stream function. Here a third possibility occurs, namely that the flow behind the shock is partly supersonic and partly subsonic, that is, a mixed flow such as is well-known for flow past a cone. This case also requires to be treated separately.

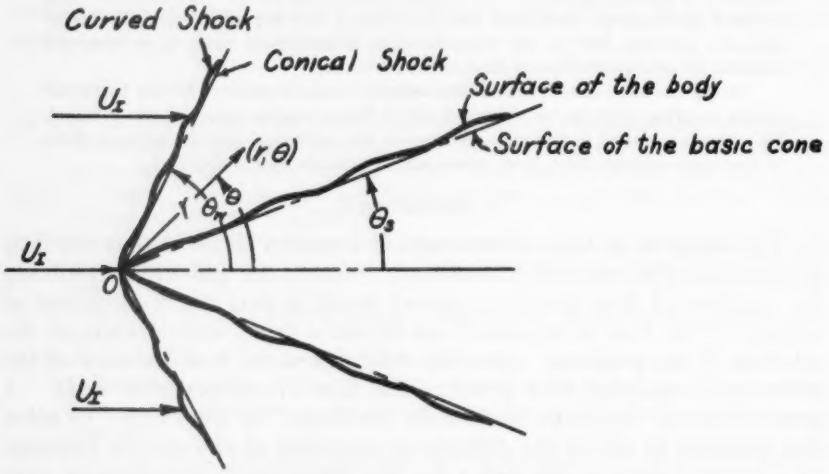

FIG. 1. Supersonic flow over body of revolution.

The basic flow in the present problem is that passing an unyawed cone which will be closely approximated to the body of revolution. We thus consider a perturbation of flow from the initial Taylor-Maccoll solution for a cone. Here it is convenient to use the polar coordinates, r, θ with the vertex of the body as the origin (Fig. 1).

The equation of the surface of the body is

$$\theta_b = \theta_s + d_1 Y(r) \tag{1}$$

where θ_b is the θ-coordinate of the surface of the body, θ_s is the semi-vertex angle of the basic cone, d_1 is a small quantity of the same order of magnitude as the perturbed stream function ψ_1 and $Y(r)$ is an arbitrary function of r which defines the shape of the body.

The location of the curved shock will be approximately on the conical surface,

$$\theta = \theta_w \tag{2}$$

where θ_w is the semi-vertex angle of the shock corresponding to the flow passing over the basic cone.

For the basic flow, let ψ_0, R_0, Θ_0 and ρ_0 be the stream function, the radial and the tangential velocity components, and the density at any point (r, θ), respectively. R_0, Θ_0 and ρ_0 are functions of θ only whose values are tabulated in Kopal's report.[3]

II. THE FUNDAMENTAL EQUATION OF THE STREAM FUNCTION

We write the total stream function ψ of the flow as follows:

$$\psi = \psi_0(r, \theta) + \psi_1(r, \theta) \tag{3}$$

where ψ_0 is the stream function of the basic flow, ψ_1 is the first order perturbation from ψ_0. Substituting Eq. 3 into the differential equation for the stream function in rotational flow and collecting terms of the same order of magnitude (see previous paper[2]) we obtain the following differential equation for ψ_1 in polar coordinates:

$$\left(1 - \frac{R_0^2}{a_0^2}\right)\frac{\partial^2\psi_1}{\partial r^2} - \frac{2R_0\Theta_0}{a_0^2}\frac{1}{r}\frac{\partial^2\psi_1}{\partial\theta\partial r} + \left(1 - \frac{\Theta_0^2}{a_0^2}\right)\frac{1}{r^2}\frac{\partial^2\psi_1}{\partial\theta^2} + F_1(\theta)\frac{1}{r^2}\frac{\partial\psi_1}{\partial\theta} + F_2(\theta)\frac{1}{r}\frac{\partial\psi_1}{\partial r} = -\rho_0 r \sin\theta\cdot\omega_1[1 + (\gamma - 1)M_0^2] + A^*F_3(\theta) \tag{4}$$

where a_0 is the local velocity of sound of the basic flow, and ω_1 is the first order vorticity.

$$A^* = \frac{2(S - S_0)}{C_v(\gamma - 1)(1 - M_0^2)}\left[1 + \frac{\gamma - 1}{2}M_0^2\right], \tag{5}$$

where S is entropy of the rotational flow, S_0 is entropy of the basic flow, C_v is specific heat at constant volume, and γ is the ratio of the specific heats.

$$M_0^2 = (R_0^2 + \Theta_0^2)/a_0^2 = q_0^2/a_0^2. \tag{6}$$

$$F_1(\theta) = \frac{4R_0\Theta_0}{a_0^2}(1 + \cos 2\theta) - \cot\theta + \frac{2\rho_0'}{\rho_0}\frac{\Theta_0 R_0}{a_0^2}\sin 2\theta - \frac{1 + \gamma}{1 - M_0^2}\frac{R_0\Theta_0}{a_0^2}\left[\frac{R_0\Theta_0}{a_0^2}\cot\theta - \frac{R_0^2}{a_0^2}\cos 2\theta - \frac{\Theta_0^2}{a_0^2}\cot\theta\sin 2\theta + \frac{\rho_0'}{\rho_0}\frac{\Theta_0}{a_0}\left(\frac{R_0}{a_0}\cos 2\theta - \frac{\Theta_0}{a_0}\sin 2\theta\right)\right]. \tag{7}$$

[3] Z. Kopal, "Tables of Supersonic Flow Around Cones," Massachusetts Institute of Technology Technical Report No. 1, 1947.

$$F_2(\theta) = -\frac{\Theta_0^2}{a_0^2} - \frac{2R_0\Theta_0}{a_0^2}(\cot\theta\cos 2\theta - \sin 2\theta) - \frac{2\rho_0'}{\rho_0}\frac{\Theta_0 R_0}{a_0^2}\cos 2\theta - \frac{1+\gamma}{1-M_0^2}\frac{R_0\Theta_0}{a_0^2}\left[\frac{R_0\Theta_0}{a_0^2} - \frac{\Theta_0^2}{a_0^2}\cot\theta\cos 2\theta - \frac{R_0^2}{a_0^2}\sin 2\theta - \frac{\rho_0'}{\rho_0}\frac{\Theta_0}{a_0}\left(\frac{R_0}{a_0}\sin 2\theta - \frac{\Theta_0}{a_0}\cos 2\theta\right)\right]. \tag{8}$$

$$F_3(\theta) = \rho_0 R_0\left(\frac{\Theta_0^2}{a_0^2}\cos\theta - \frac{R_0^2}{a_0^2}\sin\theta\right) + \frac{\Theta_0^2}{a_0^2}R_0\rho_0'\sin\theta \tag{9}$$

and

$$\rho_0' = \frac{d\rho_0}{d\theta}.$$

The nature of the differential equation (4) is determined by the value of the quantity

$$D = \frac{\Theta_0^2 R_0^2}{a_0^2} - \left(1 - \frac{R_0^2}{a_0^2}\right)\left(1 - \frac{\Theta_0^2}{a_0^2}\right) = M_0^2 - 1. \tag{10}$$

If $D > 0$, or $M_0 > 1$, Eq. 4 is a linear hyperbolic differential equation of second order. If $D < 0$, or $M_0 < 1$, Eq. 4 is a linear elliptic differential equation of second order.

From the analysis of the flow passing over an unyawed cone we know that is

$$\frac{q_{0s}^2}{q_m^2} > \frac{\gamma - 1}{\gamma + 1} \tag{11}$$

where q_{0s} is the resultant velocity on the surface of the cone and q_m is the maximum possible velocity of the flow considered, then $M_0 > 1$ throughout the region behind the shock and ahead of the basic cone and Eq. 4 is of hyperbolic type throughout this region.

If

$$\frac{q_{0w}^2}{q_m^2} < \frac{\gamma - 1}{\gamma + 1}, \tag{12}$$

where q_{0w} is the resultant velocity immediately behind the conical shock, $M_0 < 1$ throughout the region behind the shock and ahead of the basic cone and Eq. 4 will be of elliptical type throughout this region.

There is a third case where at a certain $\theta = \theta_I$ we have

$$\frac{q_0^2(\theta_I)}{q_m^2} = \frac{\gamma - 1}{\gamma + 1} \tag{13}$$

where $\theta_s < \theta_I < \theta_w$. Then Eq. 4 is of hyperbolic type when $\theta_I < \theta < \theta_w$ and of elliptical type when $\theta_s < \theta < \theta_I$.

We shall treat these three cases separately.

III. BOUNDARY CONDITIONS

The boundary condition on the surface of the body is that it is a stream surface. Thus at $\theta_b = \theta_s + d_1 Y(r)$ we have

$$\psi(r, \theta_b) = \psi_0(r, \theta_b) + \psi_1(r, \theta_b) = 0 \tag{14}$$

where

$$\psi_0 = r^2 \int_{\theta_s}^{\theta} R_0(\theta)\rho_0(\theta) \sin\theta \, d\theta. \tag{15}$$

Substituting (1) and (15) into (14) and developing in powers of d_1, we have, after neglecting the higher order terms,

$$\psi_1(r, \theta_s) = - d_1 r^2 Y(r) R_0(\theta_s)\rho_0(\theta_s) \sin\theta_s. \tag{16}$$

Equation 16 is one of the boundary conditions for $\psi_1(r, \theta)$ which is satisfied on the surface of the basic cone.

Let the shock angle with respect to the uniform stream in front of the curved shock be $\theta_w + \alpha$, where α is the perturbed shock angle. Under the assumption of a small perturbation, it is possible to find the relation between α and the perturbed velocity components R_{1w} and Θ_{1w} immediately behind the curved shock. The necessary equations are

$$R_{1w} = \alpha R_\alpha; \quad \Theta_{1w} = \alpha \Theta_\alpha \tag{17}$$

where

$$R_\alpha = \Theta_{0w}\left[1 - \frac{\rho_{0w}}{\rho_I}\right], \tag{18}$$

$$\Theta_\alpha = \frac{R_{0w}\left(1 - \dfrac{\rho_{0w}}{\rho_I}\right)\left(\dfrac{\rho_I}{\rho_{0w}} - K_{\rho w}\Theta_{0w}^2\right)}{1 + K_{\rho w}\Theta_{0w}^2} \tag{19}$$

and where the subscript w means the value immediately behind the shock, ρ_I is the density of the flow in front of the shock, and

$$K_{\rho w} = -\frac{\gamma - 1}{a_{0w}^2} \frac{\left(\dfrac{\gamma + 1}{\gamma - 1} - \dfrac{\rho_I}{\rho_{0w}}\right)\left(\dfrac{\gamma + 1}{\gamma - 1} - \dfrac{\rho_{0w}}{\rho_I}\right)}{\left(\dfrac{\gamma + 1}{\gamma - 1}\right)\left(\dfrac{\rho_{0w}}{\rho_I} + \dfrac{\rho_I}{\rho_{0w}}\right) - 2}. \tag{20}$$

Also,

$$\frac{\rho_{II} - \rho_{0w}}{\rho_{0w}} = \frac{\Delta\rho_w}{\rho_{0w}} = K_{\rho w}(R_{0w}R_{1w} + \Theta_{0w}\Theta_{1w}) \tag{21}$$

where ρ_{II} is the density of the rotational flow immediately behind the shock. Equation 21 is obtained from the energy equation. Immediately behind the shock, the perturbed velocity components obtained from the perturbed stream function at $\theta = \theta_w$ are

$$R_{1w} = \frac{(1 + K_{\rho w}\Theta_{0w}^2)\frac{1}{r}\frac{\partial \psi_1}{\partial \theta} + K_{\rho w}R_{0w}\Theta_{0w}\frac{\partial \psi_1}{\partial r}}{(1 + K_{\rho w}q_{0w}^2)\rho_{0w} r \sin \theta_w} = R_{1w}(\psi_1), \tag{22}$$

$$\Theta_{1w} = -\frac{(1 + K_{\rho w}R_{0w}^2)\frac{\partial \psi_1}{\partial r} + K_{\rho w}R_{0w}\Theta_{0w}\frac{1}{r}\frac{\partial \psi_1}{\partial \theta}}{(1 + K_{\rho w}q_{0w}^2)\rho_{0w} r \sin \theta_w} = \Theta_{1w}(\psi_1). \tag{23}$$

The second boundary condition for ψ_1 is that at $\theta = \theta_w$ the perturbed velocity components obtained from (22) and (23) are the same as those obtained from (17), that is, at

$$\alpha = \frac{R_{1w}(\psi_1)}{R_\alpha} = \frac{\Theta_{1w}(\psi_1)}{\Theta_\alpha}. \tag{24}$$

IV. ENTROPY AND VORTICITY DISTRIBUTION BEHIND THE CURVED SHOCK

After the variation of the shock angle α (24) is known, we can estimate the variation of entropy or vorticity behind the curved shock. The first order approximation to the vorticity ω is

$$\omega_1 = \rho_0 T_0 \frac{dS}{d\psi} r \sin \theta = \frac{a_0^2}{C_v \gamma(\gamma - 1)}\left[-\frac{1}{\Theta_0}\frac{\partial S}{\partial r} + \frac{1}{R_0}\frac{1}{r}\frac{\partial S}{\partial \theta}\right]. \tag{25}$$

The entropy S is a function of α, which is a function of r_w, that is, the radial distance along the conical surface $\theta = \theta_w$. Entropy is constant along a given streamline, that is, along the surface

$$r_w^2 \int_{\theta_s}^{\theta_w} R_0 \rho_0 \sin \theta \, d\theta = r^2 \int_{\theta_s}^{\theta} R_0 \rho_0 \sin \theta \, d\theta. \tag{26}$$

Furthermore,

$$\frac{\partial S}{\partial r} = \frac{dS}{d\alpha}\frac{d\alpha}{dr_w}\cdot\frac{\partial r_w}{\partial r} \tag{27a}$$

and

$$\frac{1}{r}\frac{\partial S}{\partial \theta} = \frac{dS}{d\alpha}\cdot\frac{d\alpha}{dr_w}\cdot\frac{1}{r}\frac{\partial r_w}{\partial \theta}. \tag{27b}$$

Substituting (26) and (27) into (25), we have

$$\omega_1 = \frac{a_0^2}{\gamma(\gamma - 1)}\left[\frac{1}{C_v}\frac{dS}{d\alpha}\right]\frac{d\alpha}{dr_w}\cdot\frac{r}{r_w}\frac{1}{I_R}\left[-\frac{I_R}{\Theta_0} + \frac{\rho_0 \sin \theta}{2}\right] = \omega_1(r, \theta) \tag{28}$$

where

$$I_R = \int_{\theta_s}^{\theta_w} R_0 \rho_0 \sin\theta \, d\theta,$$

$$\frac{1}{C_v}\frac{dS}{d\alpha} = \frac{\begin{array}{r}2\gamma M_1^2 \sin 2\theta_w[(\gamma-1)M_1^2\sin^2\theta_w + 2] \\ - 4\gamma \cot\theta_w[2\gamma M_1^2 \sin^2\theta_w - (\gamma-1)]\end{array}}{[(\gamma-1)M_1^2\sin^2\theta_w + 2][2\gamma M_1^2\sin^2\theta_w - (\gamma-1)]}. \quad (29)$$

M_1 = Mach number of the uniform flow in front of the shock.

$\frac{d\alpha(r_w)}{dr_w}$ is obtained from (24), since $\alpha(r_w)$ is a function of r_w only.

The entropy distribution behind the curved shock is

$$\frac{S - S_0}{C_v} \cong \left[\frac{1}{C_v}\frac{dS}{d\alpha}\right]\alpha(r_w) \quad (30)$$

where we take $S_0 = 0$.

V. SOLUTION FOR THE WEAK SHOCK CASE

When the resultant velocity of the basic flow on the surface of the cone q_{0_s} satisfies Eq. 11, the Mach number of the basic flow in the region behind the shock and over the surface of the body is always larger than one and Eq. 4 is of hyperbolic type. There are two distinct characteristic functions $\lambda_1(r, \theta)$ and $\lambda_2(r, \theta)$ of Eq. 4, that is,

$$\lambda_1 = \frac{1}{r}\frac{1}{\left(1 - \frac{R_0^2}{a_0^2}\right)}\left[\frac{R_0\Theta_0}{a_0^2} + \sqrt{M_0^2 - 1}\right] = \frac{f_1(\theta)}{r} \text{ (say)},$$

$$\lambda_2 = \frac{1}{r}\frac{1}{\left(1 - \frac{R_0^2}{a_0^2}\right)}\left[\frac{R_0\Theta_0}{a_0^2} - \sqrt{M_0^2 - 1}\right] = \frac{f_2(\theta)}{r} \text{ (say)}. \quad (31)$$

If we choose a set of new coordinates ξ and η such that

$$\xi = r\exp\left\{\int_{\theta_s}^{\theta}\frac{d\theta}{f_1(\theta)}\right\}; \quad \eta = r\exp\left\{\int_{\theta_s}^{\theta}\frac{d\theta}{f_2(\theta)}\right\}, \quad (32)$$

Eq. 4 is transformed into the canonical form

$$\frac{\partial^2\psi_1}{\partial\xi\,\partial\eta} + g_1(\xi,\eta)\frac{\partial\psi_1}{\partial\xi} + g_2(\xi,\eta)\frac{\partial\psi_1}{\partial\eta} = g_3(\xi,\eta) \quad (33)$$

where g_1, g_2 and g_3 are known functions of ξ and η; g_1 and g_2 depend only on the characteristics of the basic flow, but g_3 depends on the characteristics of the basic flow as well as ω_1 and A^*.

It should be noted that

$$r = r(\xi, \eta), \quad \theta = \theta(\xi, \eta). \tag{34}$$

On any given conical surface θ = constant, we have

$$\frac{\xi}{\eta} = K_\theta = \exp\left\{\int_{\theta_s}^{\theta}\left(\frac{1}{f_1} - \frac{1}{f_2}\right) d\theta\right\} = \text{constant} \tag{35}$$

where K_θ is a function of θ only and $K_{\theta_s} = 1$.

The boundary condition (16) may be written in the form

$$\psi_1(\xi, \eta = \xi) = -d_1\xi^2 Y(\xi) R_0(\theta_s)\rho_0(\theta_s) \sin\theta_s = \Psi(\xi) \tag{36}$$

and the boundary condition (24) as follows:
at $\eta = K_{\theta_w}\xi$

$$C_\xi \frac{\partial\psi_1(\xi, K_{\theta_w}\xi)}{\partial\xi} = C_\eta \frac{\partial\psi_1(\xi, K_{\theta_w}\xi)}{\partial\eta} \tag{37}$$

where C_ξ and C_η are known functions of the basic flow at $\theta = \theta_w$.

We are going to solve the differential equation (33) with the boundary conditions (36) and (37). Since g_3 depends on the solution of the problem, we can only solve it by successive approximation. First we solve the problem by putting $g_3 = 0$. If the result is $\psi_1 = \psi_{10}$, we then estimate $g_3(\xi, \eta)$ from this ψ_{10} by the help of (24), (28) and (30). We next solve (33) with this g_3 and the corresponding boundary conditions. This procedure can be continued until the desired accuracy is obtained.

Our problem is simply a modified Cauchy's initial value problem. Only the initial value of ψ_1 is given on the surface $\xi = \eta$, that is, on the surface of the basic cone; the first derivatives are not completely given on the surface $\xi = \eta$, since we have only one relation between the first derivatives on the surface $\xi = \eta$. In solving this problem we first express the solution of ψ_1 in terms of the unknown first derivative of ψ_1 on $\xi = \eta$ by the help of the Riemann integration method and then determine the unknown first derivative by the condition at the shock wave [(37)].

Let

$$p_1(\xi, \eta) = \frac{\partial\psi_1}{\partial\xi}, \quad q_1(\xi, \eta) = \frac{\partial\psi_1}{\partial\eta}. \tag{38}$$

On the surface $\xi = \eta$, we have

$$p_1(\xi, \xi) = \frac{d\Psi}{d\xi} - q_1(\xi, \xi). \tag{39}$$

At first $q_1(\xi, \xi)$ is unknown; we are going to express ψ_1 in terms of q_1.

By the standard Riemann integration method,[4] we can find the Riemann function $R_L(\xi, \eta; x, y)$ for the hyperbolic differential equation (33). Now the initial values on the line $\xi = \eta$ are assumed to be given (see Fig. 2). The value ψ_1 of any point P can be determined from the

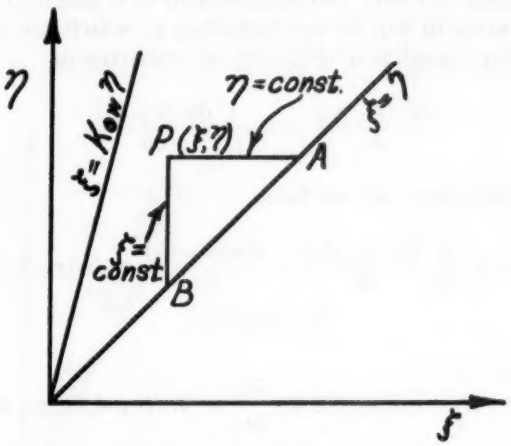

FIG. 2. Modified Cauchy initial value problem.

initial values by the following equation:

$$\psi_1(P) = \frac{\psi_{1A}R_L(A) + \psi_{1B}R_L(B)}{2} + \iint_{\nabla ABP} R_L g_3\, dx\, dy + \int_{AB}[\psi_1 R_L(-g_1\, dy + g_2\, dx) + \tfrac{1}{2}R_L\Psi'\, dx - \tfrac{1}{2}R_L q_1(dx + dy) - \tfrac{1}{2}\Psi(R_{Lx}\, dx - R_{Ly}\, dy)] \quad (40)$$

where A and B are the corresponding points on $\xi = \eta$ for P (Fig. 2); x and y replace ξ and η as variables of integration. $\Psi' = \dfrac{d\Psi}{d\xi}$, $R_{Lx} = \dfrac{\partial R_L}{\partial x}$, $R_{Ly} = \dfrac{\partial R_L}{\partial y}$. The values of ψ_1 along AB are known. [*Cf.* (6) on p. 314 of Courant and Hilbert,[4] except that there are some misprints in signs in it.]

All the terms in Eq. 40 are known except $q_1(\xi, \xi) = q_1(r)$, which is a function of r only. Symbolically Eq. 40 may be written as follows:

$$\psi_{1P}(r, \theta) = G_1(r, \theta) + G_2(r) \quad (41)$$

[4] R. Courant and D. Hilbert, "Methoden der Mathematischen Physik," Vol. 2, New York, Interscience Publishers Inc., 1937, Chapter V, pp. 311–315.

where

$$G_2(r) = -\frac{1}{2}\int_{AB} R_L q_1 (dx + dy). \tag{42}$$

$G_2(r)$ is a function of r only and independent of θ, and $G_1(r, \theta)$ represents all the other terms in Eq. 40 not including q_1, which are all known.

The boundary condition (37) may be rewritten as

$$\frac{\partial \psi_1(r_w, \theta_w)}{\partial r} = c_w \frac{1}{r_w} \frac{\partial \psi_1(r_w, \theta_w)}{\partial \theta}. \tag{43}$$

Substituting (41) into (43) we have

$$\frac{dG_2(r_w)}{dr} = c_w \frac{1}{r_w} \frac{\partial G_1(r_w, \theta_w)}{\partial \theta} - \frac{\partial G_1(r_w, \theta_w)}{\partial r} = G_0(r_w, \theta_w) \text{ (say)}. \tag{44}$$

But

$$\frac{dG_2(r_w)}{dr} = -R_L(r_B, \theta_s) q_1(r_B, \theta_s) \frac{\partial r_B}{\partial r_w} + R_L(r_A, \theta_s) q_1(r_A, \theta_s) \frac{\partial r_A}{\partial r_w}. \tag{45}$$

Since there is a linear relation between r_A, r_B and r_w (see Eq. 32, and Fig. 2), we set

$$r_A = r, \quad r_B = k_1 r_A, \quad r_w = k_2 r_A. \tag{46}$$

We have then

$$q_1(r) \frac{R_L(r, \theta_s)}{k_2} - q_1(k_1 r) \frac{R_L(k_1 r, \theta_s)}{k_2/k_1} = G_0(k_2 r, \theta_w). \tag{47}$$

The unknown first derivative $q_1(\xi, \xi) = q_1(r)$ can be determined from Eq. 47. Substituting (47) into (40) we obtain the value of ψ_1.

Thus we solve the problem when there is supersonic basic flow throughout the region between the curved shock and the surface of the body.

VI. SOLUTION FOR THE STRONG SHOCK CASE

When the resultant velocity q_{0w} of the basic flow immediately behind the conical shock satisfies (12) the Mach number of the basic flow M_0 in the region behind the shock and ahead of the body is always smaller than one and Eq. 4 is of elliptical type.

If we choose the set of new coordinates ξ_1 and η_1 such that

$$\xi_1 = \frac{\xi + \eta}{2}, \quad \eta_1 = \frac{\xi - \eta}{2i}, \tag{48}$$

$i = \sqrt{-1}$, Eq. 4 is transformed into the canonical form

$$\frac{\partial^2\psi_1}{\partial\xi_1^2} + \frac{\partial^2\psi_1}{\partial\eta_1^2} + g_4(\xi_1, \eta_1)\frac{\partial\psi_1}{\partial\xi_1} + g_5(\xi_1, \eta_1)\frac{\partial\psi_1}{\partial\eta_1} = g_6(\xi_1, \eta_1) \tag{49}$$

where

$$g_4 = 2(g_1 + g_2); \quad g_5 = \frac{2}{i}(g_1 - g_2); \quad g_6 = 4g_3.$$

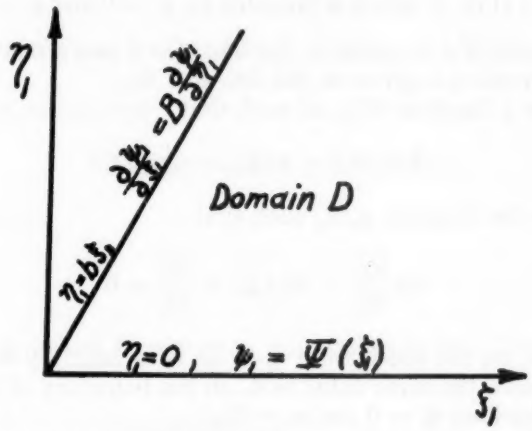

FIG. 3. Modified Dirichlet boundary value problem.

Now we have

$$r = r(\xi_1, \eta_1), \quad \theta = \theta(\xi_1, \eta_1). \tag{50}$$

On any given conical surface, θ = constant, we have

$$\xi_1 = k_\xi r, \quad \eta_1 = k_\eta r \tag{51}$$

where k_ξ and k_η are functions of θ only and at $\theta = \theta_s$, $k_{\xi s} = 1$ and $k_{\eta s} = 0$.

The boundary condition (16) may now be written as follows:

$$\psi_1(\xi_1, 0) = -d_1\xi_1^2 Y(\xi_1)R_0(\theta_s)\rho_0(\theta_s)\sin\theta_s = \Psi(\xi_1). \tag{52}$$

The boundary condition (24) may be written:

$$\frac{\partial\psi_1(\xi_{1w}, \eta_{1w})}{\partial\xi_1} = B\frac{\partial\psi_1(\xi_{1w}, \eta_{1w})}{\partial\eta_1} \tag{53}$$

where $B = i(C_\xi + C_\eta)/(C_\xi - C_\eta)$.

We are going to solve the differential equation (49) with the boundary conditions (52) and (53). Since g_6 depends on the solution of the problem, we can only solve it by successive approximation. First we solve the problem by putting $g_6 = 0$. If the result is $\psi_1 = \psi_{10}$, we then estimate g_6 from this ψ_{10} by the help of (24), (28) and (30). We next solve (49) with this g_6 and the corresponding boundary conditions. This procedure can be carried on until the desired accuracy is obtained.

Our problem is then a modified Dirichlet boundary value problem in the domain D (Fig. 3) which is bounded by $\eta_1 = 0$ and $\eta_1 = \frac{k_{\eta w}}{k_{\xi w}} \xi_1 = b\xi_1$ (say); the value of ψ_1 is given on the line $\eta_1 = 0$ and a relation between the first derivatives is given on the line $\eta_1 = b\xi_1$.

We define a function $\Phi(\xi_1, \eta_1)$ such that

$$\Phi(\xi_1, \eta_1) = \Psi(\xi_1) + \eta_1 q_2(\xi_1). \tag{54}$$

If we choose the function $q_2(\xi_1)$ such that

$$b\xi_1 \frac{dq_2}{d\xi_1} - Bq_2(\xi_1) + \frac{d\Psi}{d\xi_1} = 0 \tag{55}$$

the condition on the shock front $\eta_1 = b\xi_1$ is satisfied by Φ. The function ψ_1 will have the same value as ψ_1 on the boundary of the region D, that is, on the lines $\eta_1 = 0$ and $\eta_1 = b\xi_1$.

Now let

$$P(\xi_1, \eta_1) = \psi_1(\xi_1, \eta_1) - \Phi(\xi_1, \eta_1). \tag{56}$$

Substituting (56) into (49) we have

$$\frac{\partial^2 P}{\partial \xi_1^2} + \frac{\partial^2 P}{\partial \eta_1^2} + g_4 \frac{\partial P}{\partial \xi_1} + g_5 \frac{\partial P}{\partial \eta_1} = -\frac{\partial^2 \Phi}{\partial \xi_1^2} - g_4 \frac{\partial \Phi}{\partial \xi_1} - g_5 \frac{\partial \Phi}{\partial \eta_1} + g_6 = h(\xi_1, \eta_1) \text{ (say)} \tag{57}$$

and $P = 0$ on the boundaries $\eta_1 = 0$ and $\eta_1 = b\xi_1$.

In order to solve (57) we first find the Green's function $G(\xi_1, \eta_1; x_1, y_1)$ for the present problem. We define a function $Q(\xi_1, \eta_1; x_1, y_1)$ such that

$$\left.\begin{aligned} &\frac{\partial^2 Q}{\partial \xi_1^2} + \frac{\partial^2 Q}{\partial \eta_1^2} = 0 \text{ in } D \\ \text{and} \quad &Q = \log \frac{1}{\sqrt{(\xi_1 - x_1)^2 + (\eta_1 - y_1)^2}} \text{ on } \eta_1 = 0 \text{ and } \eta_1 = b\xi_1. \end{aligned}\right\} \tag{58}$$

Let

$$Q_1(\xi_1) = \log \frac{1}{\sqrt{(\xi_1 - x_1)^2 + y_1^2}};$$
$$Q_2(\xi_1) = \log \frac{1}{\sqrt{(\xi_1 - x_1)^2 + (b\xi_1 - y_1)^2}}. \tag{59}$$

Define a function

$$N(\xi_1, \eta_1) = Q_1(\xi_1) + \frac{\eta_1}{b\xi_1}[Q_2(\xi_1) - Q_1(\xi_1)]. \tag{60}$$

Using the transformation

$$E(\xi_1, \eta_1) = Q(\xi_1, \eta_1) - N(\xi_1, \eta_1), \tag{61}$$

Eq. 58 becomes

$$\frac{\partial^2 E}{\partial \xi_1^2} + \frac{\partial^2 E}{\partial \eta_1^2} = -\frac{\partial^2 N}{\partial \xi_1^2} \tag{62}$$

where $E = 0$ on the boundary $\eta_1 = 0$ and $\eta_1 = b\xi_1$. The solution of (62) is

$$E = -\frac{1}{\partial\pi}\int_0^\infty dx_1 \int_0^{y_1 - bx_1} \left\{ \frac{\partial^2 N(x_1, y_1)}{\partial x_1^2} \times \log \frac{1}{\sqrt{(\xi_1 - x_1)^2 + (\eta_1 - y_1)^2}} \right\} dy_1 = E(\xi_1, \eta_1). \tag{63}$$

Finally the Green's function for our problem is

$$G(\xi_1, \eta_1; x_1, y_1) = -E(\xi_1, \eta_1; x_1, y_1) - N(\xi_1, \eta_1; x_1, y_1) + \log \frac{1}{\sqrt{(\xi_1 - x_1)^2 + (\eta_1 - y_1)^2}}. \tag{64}$$

After the Green's function G is obtained, (57) can be solved by Picard's method of successive approximation, that is, we put a certain known function $\psi_{10} = \psi_1$, so that

$$P^{(0)} = \psi_{10} - \Phi,$$

then

$$P^{(1)} = -\frac{1}{2\pi}\iint_D G(\xi_1, \eta_1; x_1, y_1)\left\{ h - g_4 \frac{\partial P^{(0)}}{\partial x_1} - g_5 \frac{\partial P^{(0)}}{\partial y_1} \right\} dx_1\, dy_1 \tag{65}$$

and in general

$$P^{(n)} = -\frac{1}{2\pi}\iint_D G(\xi_1, \eta_1; x_1, y_1)\left\{h - g_4\frac{\partial P^{(n-1)}}{\partial x_1} - g_5\frac{\partial P^{(n-1)}}{\partial y_1}\right\} dx_1\, dy_1.$$

After P is found, (56) gives ψ_1. Thus we solve the problem when there is subsonic basic flow throughout the region between the curved shock and the surface of the body.

VII. SOLUTION FOR THE TRANSONIC CASE

When the resultant velocity of the basic flow satisfies (13), the local Mach number M_0 will be unity at $\theta = \theta_I$. At this point our solutions for both the supersonic case (§ V) and the subsonic case (§ VI) fail because some of the coefficients of Eq. 4 become infinite. Now we are going to investigate Eq. 4 in the neighborhood of $M_0 = 1$. Let

$$|1 - M_0^2| = \delta \tag{67}$$

where δ is a small quantity of the same order of magnitude as ψ_1. If we multiply Eq. 4 by $(1 - M_0^2)$ and neglect terms of higher order such as $(1 - M_0^2)\left(1 - \frac{R_0^2}{a_0^2}\right)\frac{\partial^2\psi_1}{\partial r^2}$, we have

$$h_1(\theta)\frac{\partial\psi_1}{\partial r} + h_2(\theta)\frac{1}{r}\frac{\partial\psi_1}{\partial\theta} = h_3(r, \theta), \tag{68}$$

where

$$h_1(\theta) = -(1+\gamma)\frac{R_0\Theta_0}{a_0^2}\left[\frac{R_0\Theta_0}{a_0^2} - \frac{\Theta_0^2}{a_0^2}\cot\theta\cos 2\theta - \frac{R_0^2}{a_0^2}\sin 2\theta - \frac{\rho_0'}{\rho_0}\frac{\Theta_0}{a_0}\left(\frac{R_0}{a_0}\sin 2\theta - \frac{\Theta_0}{a_0}\cos 2\theta\right)\right], \tag{69}$$

$$h_2(\theta) = -(1+\gamma)\frac{R_0\Theta_0}{a_0^2}\left[\frac{R_0\Theta_0}{a_0^2}\cot\theta - \frac{R_0^2}{a_0^2}\cos 2\theta - \frac{\Theta_0^2}{a_0^2}\cot\theta\sin 2\theta + \frac{\rho_0'}{\rho_0}\frac{\Theta_0}{a_0}\left(\frac{R_0}{a_0}\cos 2\theta - \frac{\Theta_0}{a_0}\sin 2\theta\right)\right], \tag{70}$$

and

$$h_3(r, \theta) = \frac{2(S - S_0)}{C_v(\gamma - 1)}\left[1 + \frac{\gamma - 1}{2}M_0^2\right]rF_3(\theta). \tag{71}$$

Equation 68 is the fundamental differential equation for ψ_1 in the transonic region.

In order to solve the problem in the transonic case we divide the whole region behind the shock front and ahead of the body into three parts. (See Fig. 4.) Let

$$\begin{aligned} \theta &= \theta_A, \quad 1 - M_0^2 = \delta, \\ \theta &= \theta_B, \quad M_0^2 - 1 = \delta. \end{aligned} \tag{72}$$

In region I, $\theta_s \leq \theta \leq \theta_A$ and the basic flow is subsonic, hence the solution for the strong shock case (§ VI) can be used; in region III $\theta_B \leq \theta \leq \theta_w$ and the basic flow is supersonic, hence the solution for the weak shock case (§ V) can be used. We join the above two solutions by means of the transonic equation (68) in the following manner:

In region I, we know the boundary values on the line $\theta = \theta_s$ but not those on $\theta = \theta_A$; furthermore $\theta = \theta_A$ is not a shock front so that we

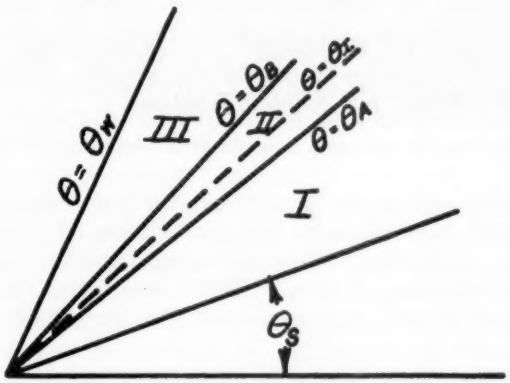

FIG. 4. Transonic flow over body of revolution.

cannot determine the boundary value on $\theta = \theta_A$ by the shock conditions (55). However we can still use the same procedure as that outlined in § VI, but we must discard the condition (55). In other words, we express the solution in region I in terms of the unknown function $q_2(\xi_1)$ which will be determined later by joining the solution in region III. Thus the boundary value on the line $\theta = \theta_A$ can be also expressed in terms of q_2 and $\Psi(\xi_1)$, the boundary value on $\theta = \theta_s$, that is,

$$\psi_1(r, \theta_A) = \Phi_1(\xi_{1A}, \eta_{1A}) = \Psi(k_{\xi A} r) + b_A k_{\xi A} r q_2(k_{\xi A} r) = \Psi_1(r). \tag{73}$$

After $\psi_1(r, \theta_A)$ is obtained, in region II, we have an initial value problem with the initial value ψ_1 given on the line $\theta = \theta_A$ and the first order linear differential equation (68). The solution for ψ_1 in region II is thus completely determined as follows:

$$\psi_1(r, \theta) = \psi_1(r, \theta_A) + \left(\frac{\partial \psi_1}{\partial \theta}\right)_{\theta=\theta_A} (\theta - \theta_A) + \left(\frac{\partial^2 \psi_1}{\partial \theta^2}\right)_{\theta=\theta_A} \frac{(\theta - \theta_A)^2}{2} + \cdots, \quad (74)$$

where $\left(\frac{\partial \psi_1}{\partial \theta}\right)_{\theta=\theta_A}, \left(\frac{\partial^2 \psi_1}{\partial \theta^2}\right)_{\theta=\theta_A}, \cdots$, etc. can be determined from (68) and (73). It should be noted that the only unknown in (74) is still q_2.

On the line $\theta = \theta_B$, we have $\psi_1(r, \theta_B)$, $\frac{\partial \psi_1(r, \theta_B)}{\partial r}$ and $\frac{1}{r} \frac{\partial \psi_1(r, \theta_B)}{\partial \theta}$ in terms of the only unknown function q_2 from (74). In region III we can use the method outlined in § V with only the following changes:

$$\xi = r \exp\left\{\int_{\theta_B}^{\theta} \frac{d\theta}{f_1}\right\}; \quad \eta = r \exp\left\{\int_{\theta_B}^{\theta} \frac{d\theta}{f_2}\right\}. \quad (75)$$

After ψ_1 in region III is obtained, we can determine the unknown function q_2 by the condition across the shock (43). As the functional form of q_2 is determined, our problem is solved.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS *

FOSDIC—A FILM OPTICAL SENSING DEVICE FOR INPUT TO COMPUTERS

An instrument that provides rapid, automatic processing of information into a form suitable for direct input to large-scale electronic computers has been developed by M. L. Greenough, H. D. Cook, M. Martens and associates of the National Bureau of Standards at the request of the Bureau of the Census. Named FOSDIC (Film Optical Sensing Device for Input to Computers), the machine reads marks on microfilmed copies of documents that have been marked with an ordinary pencil or pen, and then processes the information into electrical pulses which are recorded on magnetic tape for direct input to an electronic computer such as the Census Univac. FOSDIC is designed to reduce the work that is now involved in converting written records into a medium acceptable as input by data-processing machines. This is particularly true since FOSDIC allows considerable freedom in design of the documents and does not require the use of any special writing instrument.

It is anticipated that ultimately the use of this machine will reduce appreciably the massive amount of paper-work entailed in summarizing Census information on the entire population. Although designed for census operations, FOSDIC may be generally applied to the processing of other types of information that must be handled in large quantities.

With the development of many large-scale electronic computers in the past few years, there has been an increasing need for equipment to bridge the gap between the machines and their sources of information. This is especially true for computing systems which perform relatively little computation on a large mass of data obtained from many sources. Considerable attention has been given to computers and their input-output equipment but relatively little to "pre-input" apparatus or instrumentation permitting the computer to have direct contact with sources of information. When human beings are considered as sources of information, only two even partially automatic means of communication are in general use. These are (1) typewriters of various forms and (2) special marking instruments such as punches or conductive pencils. An alternate method is through the manual preparation of punched cards. To these methods has now been added FOSDIC, a completely automatic machine which processes marks made by an ordinary pencil or pen into a form directly usable by the computer.

* Communicated by the Director.

The method of mark sensing used by FOSDIC is the detection of specific blacked-in areas or ovals in a large field of possible answers arranged on a sheet of paper. A "yes-no" answer is given two ovals while a numerical answer is supplied with a vertical column of 10 ovals for each decade. The desired information is indicated by the locations of the marks. It then becomes the task of the sensing equipment to tell the computer precisely which ovals the enumerator has marked to signify his available information. Since FOSDIC senses the presence or absence of a mark by optical means, readings are not affected by the electrical conductivity of the mark or the paper, or by any mechanical indentation of the paper due to lack of stiffness. In practice, the interrogating agency need not supply special pencils since any common marking device is satisfactory.

The scanning process is carried out on a frame-by-frame basis. Each frame is a microfilmed picture of one side of a sheet which may be as large as 14 by 16 in. The film is placed in an optical assembly between a cathode-ray tube with a moving spot that scans the image and a photocell that produces a varying electrical signal from the light beam that has passed through the film. The current maximum capacity is about 2800 marks per sheet, since this is the present limit of adequate legibility of marks on the document. An individual film is scanned in 0.5 to 0.9 sec. Allowing for film change and other functions, the total time per frame is about $1\frac{1}{4}$secs. The average information rate is approximately 2000 binary digits or 250 decimal digits per second.

The chief problem in the design of mark sensing equipment lies in the developing of a method to locate the individual ovals with the necessary degree of precision. In a mechanically registered system, such as that used for detecting conducting-pencil marks, the pickup heads are located at fixed distances from the edge of the document. The assumptions are made that the edges are well-defined and that the paper stock has dimensional stability. In FOSDIC, however, the paper edge is replaced by a printed index mark below each column. When located by the scanning process, the index mark furnishes an exact guide to the column position. Column height, or distance from top to bottom oval, is not as critical as in a mechanical system since each answer is searched for over an area several times the size of the oval. Thus, with these degrees of freedom over ordinary scanning methods, the use of multiline documents on ordinary bond paper stock is feasible. The amount of information per document is considerably increased over punched cards.

The NBS-developed instrument is housed in four 42-in. high cabinets. In addition, there is a unit containing magnetic tape handling equipment and recording heads. The input cabinet contains the flying-spot scanning assembly. The main electronic unit generates scanning voltages which drive the cathode-ray tube beam, and receives in re-

turn the photocell signals from the scanning assembly. Answer information is fed into the output cabinet where it is converted into coded-pulse form suitable for recording on 8-channel magnetic tape. Power is supplied to all these units from the fourth cabinet.

FOSDIC's electronic equipment is composed of many separate and distinct circuit groups, each designed to carry out a unique function. For example, the Index Recognition circuit determines when the scanning beam is at the top edge of a solid mark between 0.24 and 0.36 in. high. To make this decision, however, it must previously have been informed that a number of other conditions have been met. Among these conditions are: (1) a frame to be scanned must be present, (2) the degree of tilt of the document must have been measured, (3) the scanning beam must be over the page, and (4) the apparent index mark must be genuine and not a thin vertical line crossed obliquely by the scanning beam. The reading program begins immediately after recognition of the index mark occurs.

All information is read out in serial order in blocks of 720 characters and occupies four of the eight channels on the magnetic tape. A character consists of four digits; thus each block can contain more than 2800 digits. Each document, either single or double sided, is included in one block and the remainder of the 720 characters is made up of dummy pulses. To assure that the order is not upset by the loss of a column somewhere on the page, FOSDIC makes a column count on each document. If a column is missed for any reason, such as a film defect, a characteristic record is made on the tape informing the computer that information in this block is not trustworthy. Information contained on such documents is then restored to the tape by a supplementary correction process.

Although a complete check is maintained on the information so that it cannot get out of order, this check is not as thorough nor as elaborate as a check on individual answers. Determination of overall system reliability was left to special evaluation tests in conjunction with the Univac computer. The instrument itself shows near-perfect response when operated under good marking and filming conditions. Tests with the Census computer indicate that film defects are the largest source of the few mistakes. For example, holes in the emulsion which happen to lie at an answer area leave FOSDIC with no choice but to declare an apparent mark. Although adequate statistics on the probability of these errors are not yet established, preliminary results for standard commercial processing appear to show less than one error in 100,000 ovals. If further tests indicate probable errors as infrequent as this, then it can be concluded that the transcription process through FOSDIC, including filming, does not appreciably add to other sources of error in the enumeration process.

Using electronic scanning, a high degree of flexibility can be in-

corporated into the instrument to meet or even ameliorate practical operating conditions. It offers advantages to almost all of the activities involved in the information collection process. Greater freedom in document layout and choice of size, lower cost of document paper stock, more tolerance to the weight and positioning of answer marks, and a large reduction in handling care are all obtained through the electronic scanning provided in FOSDIC. Among the more significant benefits are the use of multi-line documents several times the size of standard typewriter sheets but no thicker or stiffer, detection of rapidly made single-stroke marks of reasonable boldness, and tolerance to an appreciable amount of document mutilation and misalignment during handling and processing.

In many Census operations, microfilming has already been carried out as a routine procedure. The special problems thus introduced are the additional time for processing, the exposure control that is required, and errors due to the presence of dust and imperfections on the film. However, the use of the intermediate memory in the form of microfilm offers a number of advantages which well outweigh the problems it creates. For example, the massive paper-handling operation is separated from the scanning equipment, the mark strength is increased through the use of high-contrast films, and the electrical signal-to-noise ratio is enhanced.

EFFECT OF LIGHT ON COATED GROUNDWOOD PAPERS

Papers containing groundwood fibers, made by disintegrating logs against large grinding stones, are of considerable importance commercially. Newsprint contains about 75 per cent groundwood, and other printing papers for use in less permanent types of publications also have large percentages of this fiber.

While groundwood is one of the cheapest papermaking fibers, it is unsuited to many uses because it turns yellow very rapidly when exposed to light. However, some protection from the effects of light is afforded by a clay coating which is frequently added to achieve a better printing surface. This coating consists of very finely divided clay mixed with starch, glue, casein, or synthetic resin as a binder. In an effort to determine the degree of such protection provided to groundwood papers by commercial clay coatings, William K. Wilson and Jack L. Harvey of the National Bureau of Standards recently made a study of the effect of light on the coated papers. Their work was part of a continuing NBS program on the permanence and degradation of paper.

In addition to coated and uncoated groundwood papers, the NBS investigation included a medium-quality soda-sulfite paper and a good-quality purified sulfite-wood-pulp paper as well as samples of commercial machine-coated papers and papers made in the Bureau's experimental paper mill. Sample sheets were irradiated for eight hours on each side.

The extent of degradation was then determined by physical and chemical measurements on the specimens.

The light source was a carbon arc lamp with a cupric chloride filter and heat exchanger to remove the infrared. The filtered light was selected to correspond as closely as possible to skylight filtered by ordinary window glass, but its intensity was considerably higher. Constancy of light dosage was checked by means of a light meter.

Because the light which a paper normally receives is not sufficient to raise its temperature, and because photochemical reactions are frequently temperature-dependent, the papers were maintained at temperatures between 31° and 33° C. during irradiation. The paper was held against a water-jacketed backing plate by drawing air continuously through the sheet. This air stream passed out of the enclosed irradiation chamber in which the backing plate was mounted, circulated in a closed system over a saturated solution of sodium bromide maintained at $30.0 \pm 0.2°$ C. in a constant temperature bath, and then returned to the irradiation chamber. The bromide solution maintained the relative humidity at about 57 per cent. The water circulating through the aluminum backing was drawn from the constant temperature bath.

The degradation of the coated groundwood papers was compared with that of coated papers containing no groundwood as well as uncoated papers, with and without groundwood. It was found that a clay coating gives considerable protection against the degrading effects of light. In fact, the yellowing of the coated groundwood fibers was only about half that of the uncoated fibers of similar composition. However, groundwood is so light-sensitive that coated groundwood papers are still much less stable than are good quality papers without coating. In general, decoated papers containing groundwood reacted similarly to uncoated groundwood papers, although papers of identical fiber finish were not available for study in both coated and uncoated types.

Under the conditions of the experiments, the coated papers containing no groundwood were not degraded to a measurable extent. In contrast to the groundwood papers, degradation of uncoated papers which contained no groundwood was accompanied by bleaching.

NOTE: For further technical details, see "Effect of Light on Coated Groundwood Fibers," by William K. Wilson and Jack L. Harvey, *TAPPI*, Vol. 36, p. 459 (1953).

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

April 21, 1954

The Stated Monthly Meeting of The Franklin Institute was held on Wednesday, April 21, 1954, in the Lecture Hall, with an adjoining area being used for the overflow audience. S. Wyman Rolph, President, called the meeting to order at 8:15 P.M. Approximately 500 persons attended.

The President asked for approval of the Minutes of the February Stated Meeting, which were printed in the March JOURNAL. There being no corrections or additions, they were approved as submitted. He stated that the Minutes of the March Stated Meeting would appear in the April JOURNAL and would be submitted for approval at the next Stated Meeting.

The Secretary reported the following elections to membership for the month of March:

Active	39
Associate	29
Student	6

and a total membership, including Company and Library privileges, of 6,777 as of March 31, 1954.

The President then introduced the speaker for the evening—Dr. George L. Beers, Assistant Director of Engineering with the Radio Corporation of America. Dr. Beers discussed the principles underlying the recently adopted FCC Color Television Standards, and reviewed the progress which has been made in establishing color television as a nation-wide broadcasting service. His talk, illustrated with colored as well as black-and-white slides, was most interesting.

At 9:00 P.M. the Kraft Television Theatre presented "The Little Gods Sell Tamales," televised in color by NBC from their theatre in New York City especially for this occasion. Six RCA color receivers were located in the Lecture Hall and four in the adjoining area for the convenience of the audience. The play was a comedy folk drama about rural life in Mexico.

The meeting adjourned at 10:05 P.M. with an expression of thanks to Dr. Beers, and the Radio Corporation of America.

HENRY B. ALLEN
Secretary

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, April 14, 1954.)

HALL OF THE COMMITTEE,
PHILADELPHIA, APRIL 14, 1954.

PROFESSOR W. S. PARDOE in the Chair.

The following report was presented for final action:

No. 3275: Franklin Medal.

This report recommended the award of the Franklin Medal to Charles Edward Kenneth Mees, of Rochester, New York, "For his many outstanding contributions to the scientific knowledge of the photographic process and for his development of the first organized scientific photographic laboratory in the United States which resulted in new products advancing the usefulness of the photographic process and increasing its artistic acceptance."

JOHN FRAZER,
Secretary to Committee.

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months.:

STIPPES, M. AND R. E. BECKETT: Symmetrically Loaded Circular Plates.
GROSS, JOSEF: Nonlinear Study of Frequency Modulated Oscillator.
TAYLOR, WILLIAM C., JR.: A Neglected Method for Resolution of Polynomial Equations.
SWANN, W. F. G.: Beauty, the Ideal—of the Arts and the Sciences.
KU, Y. H. AND AHMAD KAMAL: A New Homopolar Motor.
SPRATT, H. PHILIP: An Early Marine Steam Turbine Proposal.
HESSEMER, ROBERT A., JR.: Effects of Smokes on Atmospheric Ionization.
LINHART, J. G.: The Radiation Green's Function.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, APRIL 21, 1954

ACTIVE FAMILY

James B. Brown, Jr.
Thomas J. Donaghy, Jr., D. C.
Charles H. Miller
Edward Woltemate
C. G. Ziegler

ACTIVE

Joseph E. Carney
Morrell E. Dougherty
John T. Dooley
John C. Fitzpatrick
H. Sherid Garrett
James F. Haley
Heber H. Harper
William L. Huganir
Emanuel S. Kardon
Alfred J. Laupheimer, Sr.
Alfred C. Loedding
James J. MacCarthy
Donald W. MacKensie
Stephen Malaker, Ph.D.
Frank P. McFadden
Robert M. McGredy
Harris D. McKinney
Joseph P. McLaughlin
Rodney K. Merrick
Bernard L. Miller
A. Edward Morgan
John B. Murdock
Edward C. Page, Jr.
Lazarus L. Philips
Agnes Potepan
Mrs. Thomas D. Thacher
A. Liston Townsend
George J. Volz
George E. Whitehair
Alfred Zarroli

ACTIVE NON-RESIDENT

W. J. Coppoc

NECROLOGY

C. Mason Astley, M.D. '49
Frederick Ballard '36
George Bergmann '50
George L. Coppage '13
Edwin F. Kingsbury '15
William L. McLean, Jr. '34
William Messinger, Esq. '30
Frank King Swain '36

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A.M. until 5 P.M. on Monday, Tuesday, Friday and Saturday; 2 P.M. until 10 P.M. on Wednesday and Thursday.

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NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

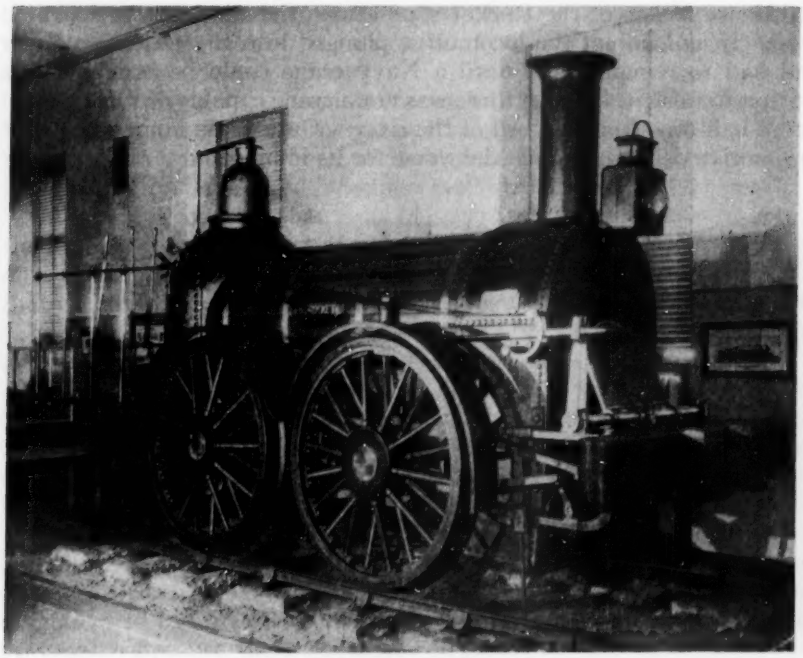
A. C. CARLTON, EXECUTIVE DIRECTOR

"THE ROCKET"—EARLY STEAM LOCOMOTIVE

Although Cugnot in France (1769) and Oliver Evans in America (1805) had demonstrated the possibility of using steam traction on land, the first railroad pioneers had horse drawn traction in mind for their purpose. Horse drawn carts were running on rails in England and America before the first steam locomotive was built.

After the famous Rainhill trails in England, in 1829, had proved that the steam locomotive was more efficient and economical than horses, the development of steam railroads was pushed energetically forward. The first locomotives used on American railroads were built in England.

The Institute Museum is the fortunate possessor of one of these old locomotives, in fact one of the very last that was imported. This was built by Braithwaite, who was the principal competitor of the Stephensons whose locomotive of the same name had won the Rainhill trials. The Reading Railroad Company had placed an order for the delivery of four locomotives to be delivered between 1838 and 1841. The Rocket



"The Rocket," 1838.

was the first to be delivered, landing at the port of Philadelphia in March of 1838. It was then trans-shipped to Reading by canal and was put to use at the opening of the Reading-Pottstown section of the line.

This small locomotive presents a startling contrast to its near neighbor the giant No. 60,000 in the Museum, the latter weighing over 350 tons to the Rocket's puny eight and a half tons. The tubular boiler is surrounded by a wooden shell, bound by iron hoops. The cylinders are horizontal, have a diameter of $10\frac{1}{2}$ inches, and a stroke of 16 inches.

In spite of their size these locomotives all had very fine service records, and were so well built that they remained in substantially their original form. The Rocket remained in service for forty-one years, during which time she had run 310,164 miles.

An additional feature of interest about this old locomotive is the piece of original track on which the locomotives ran. The rails on which trains ran were laid on hewn blocks of stone. The value of the wooden ties was not realized until the Perth-Amboy railroad, having suddenly run out of stone blocks, was forced to lay temporary wooden ties to permit the completion of a piece of track on schedule.

An interesting comment on the port facilities is provided by the experience of one of the Rocket type locomotives which was delivered here. In unloading, the locomotive plunged into the Delaware River and had to remain there until a Navy crane could be brought from Boston to raise it, because there was not a crane capable of raising eight and a half tons from the bed of the river. Once again on dry land, the locomotive performed none the worse for its immersion.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

THE SIMILARITY OF CANCER GLYCOLYSIS AND YEAST FERMENTATION

BY

ELLICE McDONALD

A systematic search has been under way in our laboratory for inhibitors of carbohydrate metabolism which might be effective in the treatment of cancer. The yeast cell was selected as one of the screening systems because its carbohydrate metabolism is in many respects similar to that of the cancer cell.

About 30 years ago, Warburg (1) discovered that cancer tissue rapidly converts glucose to lactic acid. This process of metabolizing glucose is known as glycolysis. He devised a sensitive manometric method for measuring the acid production and other metabolic activities of a small slice of tumor. Further work showed that cancer tissue produces a large quantity of lactic acid not only in the test tube but also in the animal body. The amount of lactic acid which appears in the blood flowing from a tumor nodule is almost exactly equal to the decrease in the glucose content.

The glycolysis of cancer tissue is not limited to anaerobic conditions. When oxygen is present, the rate of tumor glycolysis is only slightly reduced. In contrast, the glycolysis of most normal tissues, such as brain and muscle, is strongly suppressed in the presence of oxygen.

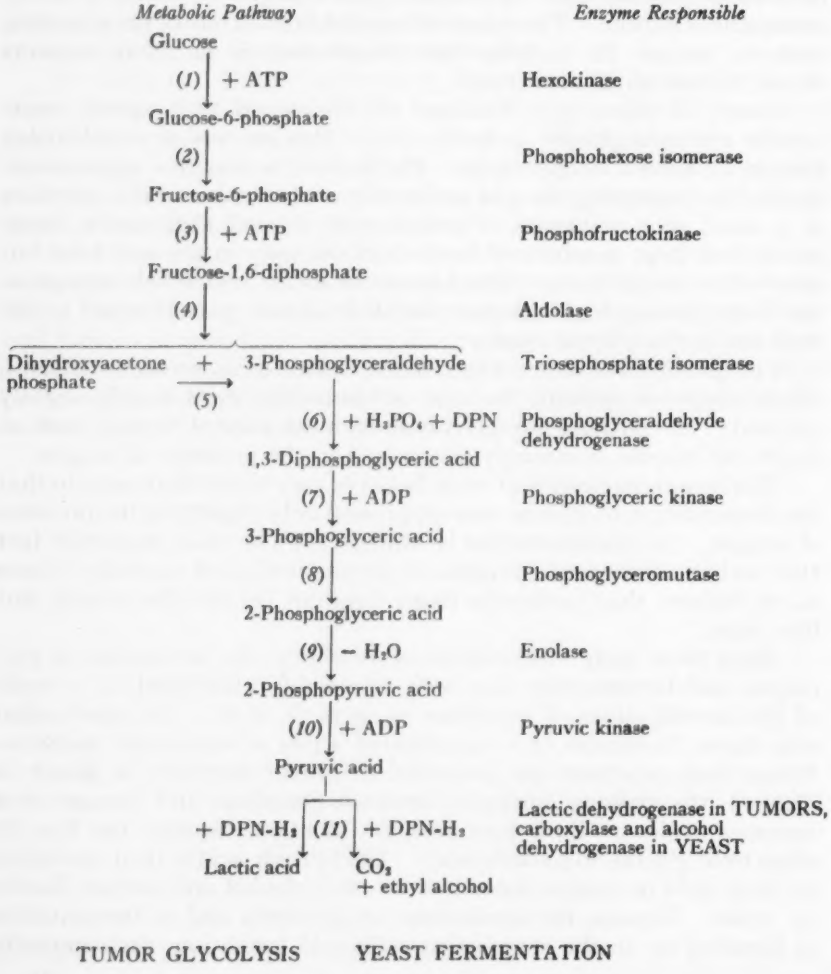
Warburg recognized that yeast behaves very much like cancer in that the fermentation of glucose was suppressed only slightly in the presence of oxygen. In this connection he stated (2) "The most important fact that we have discovered in regard to the metabolism of carcinoma tissue is, we believe, that carcinoma tissue does not behave like muscle, but like yeast."

Since these early observations of Warburg, the mechanism of glycolysis and fermentation has been thoroughly elucidated as a result of the investigations of numerous workers (3, 4, 5). The mechanism was shown to consist of a complicated series of enzymatic reactions. When these reactions are presented in proper sequence as shown in Table I, the striking similarity between glycolysis and fermentation becomes evident. Both processes are identical through the first 10 steps from glucose to pyruvic acid. The pyruvic acid is then converted to lactic acid by cancer tissue and to ethyl alcohol and carbon dioxide by yeast. Because the mechanism of glycolysis and of fermentation is identical up to the stage of pyruvic acid formation, the composite

scheme is usually referred to as the Meyerhof-Embden pathway. The metabolism of glucose has been found to follow this pathway throughout the domain of living organisms from bacteria and fungi to higher plants and animals. The degree to which this system functions varies from one type of cell to another. However, the rate is high in all malignant tumor cells.

The Meyerhof-Embden pathway is actually cyclical in nature. Adenosine triphosphate (ATP), which contains high energy phosphate

TABLE I.—*The Meyerhof-Embden Pathway of Tumor Glycolysis and Yeast Fermentation*



bonds, is formed in reactions 7 and 10 of Table I. It is utilized in the phosphorylation of glucose (step 1) and of fructose-6-phosphate (step 3). When one molecule of glucose is metabolized, 4 molecules of ATP are formed, and only 2 molecules are decomposed to adenosine diphosphate (ADP), giving a net production of 2 ATP molecules. In the growing cell, the ATP formed by the glycolytic mechanism is available as a source of energy for the synthesis of proteins, nucleic acid, and other cell constituents.

The maintenance of the cycle is dependent not only on the ATP balance discussed above but also upon the balanced oxidation and reduction of diphosphopyridine nucleotide (DPN). DPN is reduced in reaction 6 and reoxidized in reaction 11 by both tumor and yeast.

Since the pathway for yeast fermentation and tumor glycolysis is identical except at the last stage, it is highly probable that an inhibitor of any step of yeast fermentation except the last would also be an inhibitor of tumor glycolysis. In screening materials for an inhibitor of this process, yeast offers certain advantages over tumor tissue. The technic devised by Warburg requires thin slices of tumor tissue for the study of glycolysis. Ordinarily such tumor slices are not uniform with respect to the proportion of tumor cells. A typical solid tumor is composed of not only malignant cells but also normal blood capillaries, connective tissue, and dying or dead cells. For these reasons it is difficult to obtain exact duplication of results with tumor slices. Yeast, on the other hand, is readily available and homogeneous, reproducible cell suspensions are readily obtained. Yeast is an ideal test material, therefore, when numerous compounds are to be tested as possible inhibitors of tumor glycolysis, and has been adopted for this purpose here at the Biochemical Research Foundation. The suitability of yeast as the test system has been confirmed by the fact that the most potent inhibitors of fermentation have also been found effective as inhibitors of tumor glycolysis.

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NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

NICOL H. SMITH, DIRECTOR

A DESIGN FOR COMFORTABLE WORKING CONDITIONS

BY

A. DONALD HAY

Recent improvements in air conditioning techniques and equipment have raised the comfort standards demanded by the public. If an installation is properly engineered the equipment should pay for itself in a very few years and create a comfortable environment over a much longer period.

The author, Senior Staff Engineer in the Analysis Section acting as Consulting Engineer, has designed a number of successful air conditioning installations. One design which produced a most satisfactory office area will be described.

The McLean Engineering Laboratories in Princeton, N. J., which specializes in air moving equipment, wished to air condition their new office area by this means alone to demonstrate their products. Objectives of the design were to create a nearly uniform air temperature and mean radiant temperature in the area in the winter, and a comfortable air temperature and relative humidity during the summer.

Figure 1 shows a vertical section through the office wing. The floor is of modified slab construction. Above eight inches of gravel was placed a vapor barrier and rigid fiber-glass insulation. The entire floor area (except above the supply duct) was then covered with 4×12×12-in.

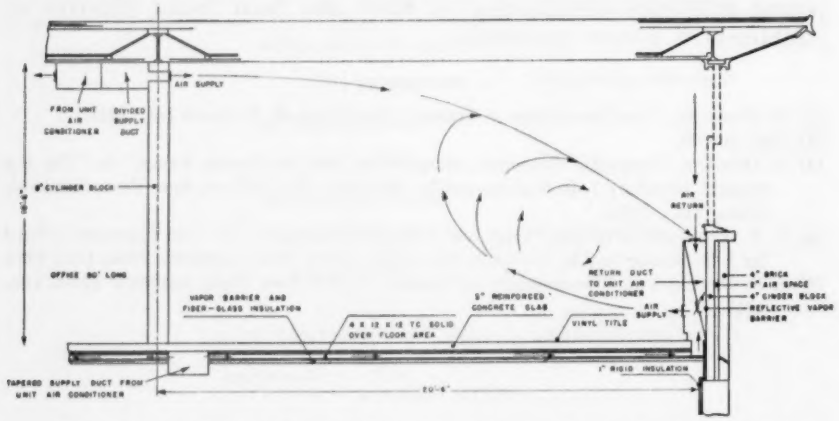

FIG. 1. Section through office showing supply and return ducts and movement of air.

FIG. 2. View of office with system installed.

tiles, so laid that air passages were aligned from the tapered supply duct, which ran beneath the floor at the center of the room, to the bustle beneath the windows. Above the tile was placed waterproof paper and then three inches of reinforced concrete dressed with vinyl tile. Several experimental installations have been made which use hollow reinforced pre-cast concrete beams in place of the tile, but the terra cotta tile employed in this case is far cheaper and gives better heat transfer. The tile floor reaches a temperature of from 80° to 85° F. in cold weather, producing comfortable conditions at and near the floor.

The office wing has continuous double hung windows above the brick section of the walls. In order to eliminate the cold wall effect from the brick section, to counteract the radiant heat lost through the windows, and to withdraw the cold drafts from the windows, a continuous plywood bustle was built on the inside of the room beneath the windows. In the winter, warm air enters one half of this section from the passages beneath the floor, warms the plywood, and leaves the section through evenly spaced grilles leading either into the working area or into the return duct or both. As presently operated, the installation directs all of the supply air into the working area. All of these grilles employ adjustable dampers. The other half of the bustle houses the return duct which returns the air from beneath the windows to the conditioning unit. Long narrow grilles beneath each window provide an almost continuous opening to catch cold drafts.

If a large volume of cooled air were sent through the passages in the floor during hot humid weather, condensation on the floor and cooled

wall would be likely. For this reason, and to create better air movement throughout the year, a second supply duct was placed along the ceiling near the center of the office wing. This duct discharges either heated or cooled air to both halves of the area. Because the supply duct beneath the floor is essentially closed during the cooling operation, the discharge velocity from the overhead grilles is automatically increased—a desirable condition. The overhead duct is divided so that separate thermostats can control separate dampers which separately control the temperature in the two halves of the area. All of the air returns to the conditioner through the return ducts beneath the windows to a packaged air conditioning unit located in the adjacent shop wing.

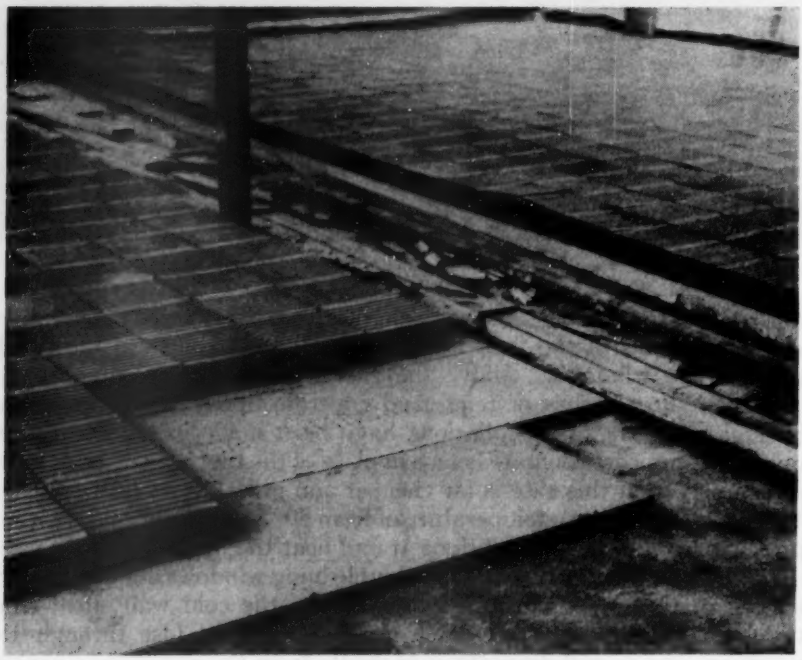

FIG. 3. Composition of floor showing air ducts.

Conditioned air enters the supply ducts at temperatures up to 120°F. Air slowly leaves the supply registers beneath the windows at temperatures up to 90° F. The dry bulb temperature of the air in the room seldom varies by more than two degrees Fahrenheit from the floor to the ceiling and from the thermostat to the farthest point in the room. The radiant heat from the floor and the outer wall counteracts chilling effects from the cold surfaces in the winter, and cold drafts are eliminated at their source. The resulting comfort during cold weather is marked. The careful distribution and removal of air during the cooling season yields optimum comfort during this period also.

BOOK REVIEWS

SCIENTIFIC PAPERS: PRESENTED TO MAX BORN (on his Retirement from the Tait Chair of Natural Philosophy in the University of Edinburgh). Contributors: Sir Edward Appleton, David Bohm, Louis de Broglie, Richard Courant, Albert Einstein, Pascual Jordan, Th. V. Kármán and S. S. Penner, Alfred Landé, Erwin Schrödinger, Hermann Weyl. 94 pages, portrait, 15 × 23 cm. New York, Hafner Publishing Company, [1953]. Price, $2.50.

The purpose of this book is expressed in the first paragraph of the Foreword, as follows:

> This modest volume of essays, written by a few of his friends throughout the world, and sponsored by colleagues and well-wishers in Edinburgh and elsewhere, is offered as a tribute to Max Born as he relinquishes the Tait Chair of Natural Philosophy in the University of Edinburgh. During the seventeen years of his tenure of this chair, he has brought distinction to the University, both by his own researches and through the influence of the school of theoretical physics which he established here.

The book contains papers by eleven distinguished investigators. It starts with an article "Geomagnetism and the Ionosphere" by Sir Edward Appleton, an article which should be of considerable value to those making their first acquaintance with the field.

Six of the contributions center on discussions of statistical matters, particularly in relation to the interpretation of the quantum theory. They are rendered none-the-less interesting by the fact that the authors do not always see eye to eye on these matters. The general reader will welcome these papers because their very compactness has forced the authors to dissect in simple form the subjects of their discussions from the totality of theorems and arguments in this field, where often some of these points lie embedded and partially obscured by such a conglomeration of analysis as to hide very vital matters. It is refreshing to see that these matters caused some concern to the authors, in spite of the fact that they are regarded with equanimity by many lesser minds.

The participants in this discussion are David Bohm, Louis de Broglie, Albert Einstein, Pascual Jordan, Alfred Landé, and Erwin Schrödinger.

Finally we have a communication by Richard Courant, "On the Classification of Partial Differential Equations," one by Th. v. Kármán and S. S. Penner on "The Thermal Theory of Constant-Pressure Deflagration for First-Order Global Reactions," and an article by Hermann Weyl entitled "A Simple Example for the Legitimate Passage from Complex Numbers to Numbers of an Arbitrary Field." W. F. G. SWANN

ELEMENTS OF STATISTICS, by H. C. Fryer. 262 pages, diagrams, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1954. Price, $4.75.

According to the author "this book has been developed to provide the basis of an introductory course in probability and statistics for the college and university student." The only background needed is a knowledge of college algebra.

The book will probably serve well in such a course. It is subdivided into seven chapters: 1. History and Introduction. 2. The Summarization of Sets of Data Involving One Type of Measurement. 3. Elementary Probability. 4. The Binomial and Normal Frequency Distributions. 5. Sampling from Binomial Populations. 6. Introductory Sampling Theory for a Normal Population Involving Only One Variable. 7. Linear Regression and Correlation.

The concept of sampling is well developed from mathematical models of various populations with emphasis on the importance of sampling variations. We find also a thorough discussion of confidence intervals and

the statistical test of hypotheses. Throughout the book, the author has adopted an informal manner of presentation providing enough exposition to insure the clarity of the concepts. At several places, brief but sufficient mention is made of relatively recent developments. For example, we find the use of the sample range instead of the standard deviation in certain tests of statistical hypotheses in the sixth chapter. A short note on rank correlation with reference to Kendall's Tau Coefficient concludes the seventh chapter.

Only the last chapter appears somewhat sketchy. In addition, a number of diagrams are not drawn exactly to the scale shown and may confuse the student. As a whole, however, the book covers the introductory material quite thoroughly. An unusually large number of examples are presented from such fields as genetics, engineering, chemistry, biology, and many others. Actual sampling distributions aid the reader's empirical experience. Therefore, this reviewer can recommend the book to statistically minded novices in practically all fields of research.

CARL HAMMER

CONSTRUCTION METHODS AND MACHINERY, by F. H. Kellogg. 415 pages, illustrations, 15 × 22 cm. New York, Prentice-Hall, Inc., 1954. Price, $10.00.

The second largest industry in the country, construction, is more of an art than a science. Only in relatively recent years have modern production methods been applied to this industry. Mr. Kellogg, Dean of the Engineering School at the University of Mississippi, in the first important book on the subject in ten years, has gathered together all the latest methods of operating and presented them in his book *Construction Methods and Machinery*. His wide practical experience in construction of dams, highways, railways, bridges and earthworks, combined with a number of years of teaching experience, is reflected in the material in the book and in its logical arrangement.

Construction Methods and Machinery is divided into three parts. Part I concerns operational management. Here Dean Kellogg properly begins his treatise by discussing the organization of the staff to execute a construction job, the preliminary and detail planning, the control of production, and the economics of construction equipment. Part II concerns all the machinery that is involved from that required to produce the source of power to various means of transportation and to hoisting, conveying and pumping. In Part III construction methods are described. This is the most extensive section of the book. The preparation of a site is first explained, beginning with the clearing of brush and trees to the final grading and drainage. In the course of the discussion, the use of such mechanical equipment as tractor and 'dozer, winch and cable, drag lines and chain saws is explained. Various methods of earth and rock excavation, and such foundation treatment as soil borings, pile driving, sheet piling and cofferdams are described. The preparation of concrete and its use is relatively briefly but adequately discussed. The book concludes with descriptions of wood, steel and masonry construction. Problems illustrate the application of the principles presented in each chapter.

E. W. HAMMER

PROBABILITY AND INFORMATION THEORY, WITH APPLICATIONS TO RADAR, by P. M. Woodward. 128 pages, 15 × 22 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $4.50.

This little monograph is a very concise exposition of Shannon's mathematical theory of communication, more popularly known as "Information Theory." It is written in a lucid and lively style.

The book is divided into three major parts. The first part gives the essential background material in probability theory; the middle portion deals with information theory; and the final portion is devoted to the application of information theory to the radar problem. Thus the first part of the book develops probability theory from scratch, presents the concept of entropy, describes the waveform analysis of signals and the sampling theorem, and develops the idea of Gaussian noise. This is followed by a discussion of the theory of information, information content, channel capacity of a noisy communication channel, and information-destroying processes. Finally, the major part of the book is devoted

to the radar problem and in particular to the problem of detection of the radar signal, the measurement of range and radial velocity.

The problem of discrimination of range is a problem in discrimination of time delays, while the problem of radial velocity measurement is a problem in frequency discrimination. Hence the problems considered in this book are the one-dimensional radar problems of signal detection, and discrimination of time and frequency. These are exactly the problems of telephone and radio communication. Therefore, the field of interest of this book is much wider than the word "radar" in the title indicates.

The first flurry of enthusiasm that occurred when Shannon's papers made their appearance has dissipated itself. The probable causes for the present decline in enthusiasm in this field are due to the fact that very little has been done in extending Shannon's results, and also due to the fact that information theory has not turned out to be a panacea for all the problems in the communication field. This book should go a long way in convincing the skeptics that information theory is not yet dead.

H. L. PLATZER

NUCLEAR MOMENTS, by Norman F. Ramsey. 169 pages, diagrams, 15 × 23 cm. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1953. Price, $5.00.

The first four sections of this small book first appeared in Volume I of "Experimental Nuclear Physics," edited by E. Segrè. The new material is mainly contained in a longish fifth section which is devoted to a discussion of applications to chemistry and solid state physics. With the addition of an appendix on nuclear shell structure, new references, and author and subject indexes, this monograph has become a useful addition to the scientific literature.

There seem to have been two main reasons for reprinting material which is readily available in a standard reference such as "Experimental Nuclear Physics." First is the desire of those who apply nuclear moment measurements to have the added section on applications.* Second is the high price of the Segrè book.

* The first four sections are primarily nuclear physics.

This book is testimony to the large growth in the field of nuclear moment measurements which has occurred since 1945. Since that time new and improved techniques, including microwave methods, have enabled the nuclear scientist to gain important information. Not only are these measurements of interest to the nuclear physicist, but nuclear moment measurements "yield data concerning the nature of molecular bonds, locations of atoms in molecules, electron distributions in molecules, the shape of vibrational potential in molecules, molecular association and dissociation in liquids, chemical exchange, excitation of hindered rotations in solids, strains in solids, crystal structure, electron distributions in metals, and many other important chemical and solid state properties." Nuclear moment measurements provide quite a tool.

Following a brief introduction, the author develops the necessary theory in the section on Interaction of a Nucleus with Atomic and Molecular Fields. This is followed by sections on Experimental Methods of Measuring Nuclear Moments and Statistics, Results of Measurements of Nuclear Moments and Statistics, and Applications to Chemistry and Solid State Physics.

The author has contributed much to the growth of this field for he is cited no less than 18 times in the 17 pages of references. He has given us a well-written book which will serve both as a source and as a guide to the literature. LEONARD MULDAWER

STATICS AND STRENGTH OF MATERIALS, by Roland H. Trathen. 506 pages, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1954. Price, $7.50.

Somewhat different from the usual text on strength of materials because it also combines the subject matter with statics, *Statics and Strength of Materials* is nevertheless a fairly good book on strength of materials. Whether it is improved any by treating statics concurrently is hard to say; probably it makes very little difference. Statics is a subdivision of mechanics and is usually treated under that subject. But, since strength of materials is based upon statics, it is logical to include statics with it. And it does demonstrate its practical

use in engineering more positively than if treated separately.

Probably more technical books are written on the strength of materials than on any other field of engineering. In such books one expects to find the fundamental concepts reviewed and the treatment of such subjects as trusses, connections, stresses and stress-strain relations, determinate and indeterminate beams and structures, theories of failure, deflection of beams, columns and dynamic loadings. All these are in *Statics and Strength of Materials.* While some of these subjects might have been discussed more deeply, it should be remembered that this is an elementary book. However, much of the material, although elementary, is treated in detail. Each step in the derivation of many of the equations is shown, so that one can follow the development quite easily. In an introductory book, this is very helpful, and, of course, makes the book more useful.

E. W. HAMMER

TECHNICAL ASPECTS OF SOUND, edited by E. G. Richardson. 544 pages, illustrations, 16 × 24 cm. Houston, Elsevier Press, 1953. Price, $11.00.

This book, the first of two volumes which will bear the above title, has the subtitle "Sonic Range and Airborne Sound." The second volume which is to appear in 1954 will deal with the ultrasonic range and applications, including underwater propagation.

The list of contributors to this book is impressive and is compiled from both sides of the Atlantic. Designed to be a source book, the introductory chapter contains a brief resume of that portion of the theory of sound which is basic to all the subjects covered in the book. Additional theory is presented in the individual chapters which are grouped into the following six divisions: Acoustic Measurements and Materials; Acoustics of Buildings; Noise; Speech and Hearing; Sound Reproduction; Analysis and Synthesis of Sound-Design; and Performance of Musical Instruments.

As stated in the preface, this book was designed to provide a means whereby "both research worker, industrialist and advanced student shall be able to survey, at a fairly advanced level, the whole of the applications which this rapidly advancing branch of physics has received in the past few decades." Since this book purports to cover all topics in the sonic range, the discussion is necessarily very brief. For example, under the heading of Measurements in the chapter on microphones, one finds the following statement: "This important subject, too extensive for inclusion here, is surveyed in detail by Beranek." Then a reference is given to the appropriate work. While not many subjects are covered in such a cursory fashion, it is felt that this book will find its greatest use as a guide to the literature. References are grouped together at the end of each chapter and are quite extensive. If this is to be the primary use for this book, then it is unfortunate that the index is not more complete. An author index for the references cited would also contribute a great deal.

The material in the book is well arranged and clearly written and should accomplish the purpose for which it was designed.

RICHARD STAVSETH

MR. TOMPKINS LEARNS THE FACTS OF LIFE, by George Gamow. 88 pages, drawings, 18 × 22 cm. New York, Cambridge University Press, 1953. Price, $2.75.

George Gamow has had singular success in taking an abstruse topic and telling its story in clear, everyday, understandable language. As a result, his mythical Mr. Tompkins has been able to portray vividly experiences in the field of relativity and in the inside of the atom. The adventures were described in a fashion which insures reader interest.

However, in *Mr. Tompkins Learns the Facts of Life* Dr. Gamow has stepped out of his class. When he writes of the subjects he knows intimately such as relativity, atomics, cosmogony, Dr. Gamow is one of the finest expositors in the country. But Dr. Gamow does not possess this remarkable intimacy with biology and the book is not up to the standard we expect.

The book comprises four chapters dealing with the blood stream, the chromosomes and genes (the thread of life), the brain and finally a philosophical discourse on the nature of life.

Some of the writing, especially on the brain and computers, makes fascinating read-

ing. The last chapter on the Nature of Life could have been materially lengthened to add significant points which were not made. The universality of carbon and its ability to combine with other elements, the apparently uniform distribution of the elements, the infinity of places in the universe where life may be found are just a few of the omissions in this chapter.

While this book is not up to the standard expected of Dr. Gamow, it is still a highly readable one and it most assuredly will not affect the output of the author, whose prolific writings will eventually class him as a scientific Edgar Rice Burroughs.

I. M. LEVITT

ESSENTIALS OF ENGINEERING THERMODYNAMICS, by Herman J. Stoever. 279 pages, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $4.50.

Several years ago Professor Stoever prepared a book called "Engineering Thermodynamics" which was a fairly comprehensive treatment of thermodynamics. *Essentials of Engineering Thermodynamics,* by the same author (Professor of Mechanical Engineering at Iowa State College), is a condensed version of his first book, in order that it may be used for a one-semester elementary course.

Like its predecessor, *Essentials of Engineering Thermodynamics* deals with that branch of thermodynamics concerned with mechanical work. It does not deal with electrical or magnetic work, or with substances that change chemically. As before, too, the book is divided into three parts. Part I, the first law of thermodynamics, is concerned with such basic properties of substances as pressure, volume, temperature; mechanical work and heat; internal energy and enthalpy; and the analysis of simple processes. Part II, the second law of thermodynamics, is concerned with power cycles and refrigeration cycles, and with entropy. Part III, applications, is concerned with the application of the principles developed in the first two parts to practical industrial problems. In this last part, Professor Stoever illustrates the engineering applications of thermodynamics in the studies of the performance of steam power plants, internal combustion engines, refrigeration plants, air compressors, nozzles and air conditioning.

A condensed version of a book necessarily omits some material which may not be desirable. However, a condensation, if well executed, may be beneficial in the case of a textbook because all extraneous material is eliminated and the student can be concerned only with the significant material. He is not distracted. And this is the case with *Essentials of Engineering Thermodynamics;* it is a good, short, concise book on the fundamentals of thermodynamics.

E. W. HAMMER

THE SUN. VOLUME I, THE SOLAR SYSTEM, edited by Gerard P. Kuiper. 745 pages, plates, illustrations, 18 × 25 cm. Chicago, The University of Chicago Press, 1953. Price, $12.50.

One of the most ambitious attempts to collect and systematize all the significant knowledge of the solar system has made an auspicious start in the publication of *The Sun.* This is the first of a series of four books detailing the solar system.

The only place where there appears a systematized account of this field is in the *Handbuch der Astrophysik* and that is almost 25 years old. The tremendous advances in solar and planetary studies from the mid-thirties to the present has rendered these works somewhat obsolete so there is a prime place for this new contribution.

The authors come from six countries and are masters of their respective fields. The book is dedicated to the late Bernard Lyot—the man who was to have been a major contributor to this volume. The dedicatee, a Franklin Institute medalist, had done more to advance our knowledge of the sun than any other man in astronomy and throughout the entire book one finds the skillful imprint of this remarkable scientist.

On the dust cover the editor claims ". . . it is designed to be a reference handbook for the specialist. On the other hand, it is written so as to be intelligible to anyone versed in the physical sciences." That the book is designed for the specialist there is no doubt but because it is a truly significant

contribution in the field indicates that it will not necessarily be intelligible to anyone versed in the physical sciences. However, it is a highly technical, comprehensive and authoritative work which must be in the library of the astronomer.

The Sun begins with an introduction by Dr. Leo Goldberg. Here is contained an excellent historical section which sets the stage for the succeeding chapters. This chapter is remarkable for its completeness. It ranges from sunspot discovery to hydrodynamic problems associated with the sun, from eclipses to solar-terrestrial relations.

Chapter II, by Dr. Bengt Stromgren, treats the sun as a star. The application of the theory of stellar interiors is applied to the sun to yield detailed physical and chemical information of the solar structure. In this section are found the thermonuclear processes leading to the production of the sun's energy with a detailed discussion of the *proton-proton* reaction and the *carbon cycle.*

In Chapter III is detailed our knowledge of the source of practically the entire observed solar radiation—the photosphere. Limb darkening, polarization, formation of spectral lines and the theory underlying this formation and the explanation of line characteristics are covered in this section by Dr. M. Minnaert.

Dr. Charlotte E. Moore contributes Chapter IV on the identification of solar lines. Therein is contained the most recent summary of elements in the sun. This facet of solar research is one of the most important in the field. Upon the complete and accurate identification of the spectral lines depends the quantitative study of the sun.

The chromosphere and the corona comprise Chapter V by H. C. Van de Hulst. The temperature, chemical composition, density, heating, support and pressure are as far as possible detailed. Dr. Van de Hulst shows that a good bit of our information is incomplete and in some respects uncertain, for which reason our knowledge of the chromosphere is incomplete. The treatment of the corona is as complete as can be realized at this time. The curious phenomenon of coronal temperature is investigated thoroughly and the four independent lines of evidence evaluated. In every case the temperature appears to exceed a million degrees.

The longest chapter of the book, by Dr. K. O. Kilpenheuer, deals with solar activity. In this section sunspots, faculae, chromospheric flares, prominences, corpuscular emission and cosmic rays are treated. It is one of the finest examples of exposition this writer has encountered.

The book contains chapters on solar radio emission, solar electrodynamics, empirical problems and equipment. The last chapter is a contribution of 14 writers and covers every facet in the solar instrumentation field.

Through the entire book there appears a sense of integration which is neither haphazard nor happenstance. To Dr. Kuiper must go the credit for the lucid, coherent and precise editing which graces this work. He has adeptly coordinated the various patterns into a complete picture of the nature of our sun.

It is easy to properly assess this book at this time for this reviewer is convinced that it will maintain its position as one of the most significant contributions in astronomy for some time in the future.

I. M. LEVITT

COMPLEX VARIABLE THEORY & TRANSFORM CALCULUS, by N. W. McLachlan. Second edition, 388 pages, 15 × 22 cm. New York, Cambridge University Press, 1953. Price, $10.00.

This is a revised and slightly enlarged edition of the book *Complex Variable and Operational Calculus with Technical Applications,* first published in 1939 and reprinted in 1944.

The general disposition of the book has been retained: Part I (pages 3 to 131) deals with the theory of functions of a complex variable to the extent required for the evaluation of contour integrals associated with the inverse Laplace transformation. The subject is introduced in the conventional manner: analytic functions, poles, essential singularities, branch points, Cauchy's theorem, Taylor and Laurent expansions, analytic continuation, calculus of residues, evaluation of contour integrals. Next, the

Bromwich-Wagner contour and its equivalents are discussed and the problem of approximate contour integration is considered in some detail. A brief section is devoted to some properties of the Gamma, Error and Bessel functions required in the sequel.

In Part II (pages 135 to 191), the "p-multiplied Laplace transform" is defined and some of the more common operational relations are derived. Their application in the solution of linear differential equations with constant coefficients is illustrated. The section on discontinuous functions includes some new topics: sequences of unit impulses, staircase functions. The relationship between the p-multiplied Laplace transform and the Fourier transform of the same function is discussed in some detail.

Some material of the first two parts has been relegated to appendices (pages 337–360). A short list of p-multiplied Laplace transforms is given on pages 361 to 365.

Part III (pages 195 to 334) is devoted to worked examples of applications of the transform method to a large variety of engineering problems. The last section contains 102 examples to be worked by the reader (with answers). A list of 273 references (up to 1951) is given at the end of the book.

In all, the second edition is a worthy successor of the first which has played an important role in the dissemination of operational methods among engineers. Its principal merit lies in the profusion of carefully worked-out examples giving a step-by-step account of the method of solution appropriate to the problem at hand.

This reviewer feels that its usefulness as a textbook would be enhanced by a change of the order of presentation of the subject matter: from the didactic point of view, it appears preferable to introduce first the formal aspect of the Laplace transformation and to let the student acquire a facility in solving problems by the use of a table of Laplace transforms. Only then the inversion integral should be introduced and the theory of the methods underlying its evaluation be presented in greater detail.

Why not use the non-"p-multiplied Laplace transformation," that is, the plain Laplace transformation in the third edition?

OTTO R. SPIES

BOOK NOTES

REVIEW OF TEXTILE PROGRESS. VOLUME 4. 1952. 560 pages, 14 × 21 cm. Manchester (England), The Textile Institute and the Society of Dyers & Colourists, 1953. Price, $5.00.

The fourth volume in this useful series covers advances in the textile field during 1952, in which production was reduced but research on new processes was increased. New subsections in this volume cover carpet manufacture and the production of regenerated protein fibers, while the sections on cotton production and felt have been omitted from the usual listing of subjects. This authoritative volume, prepared by experts, should be in the library of all textile technologists, students and libraries.

MECHANICS OF MATERIALS, by Philip G. Laurson and William J. Cox. Third edition, 414 pages, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1954. Price, $5.75.

The most important changes in this third edition are the increased emphasis on the statics of stress determination, and the postponement of the material on stresses determined by deformation. The tables of structural shapes have been brought into conformity with present day standards, and many new problems have been added. In its revised form, this text will continue to be of real service both to students and practicing engineers.

DIRECT AND ALTERNATING CURRENTS. THEORY AND MACHINERY, by E. A. Loew, assisted by F. R. Bergseth. Fourth edition, 637 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $7.50.

The authors have brought up-to-date all the material in the new fourth edition of this well known text. The chapters on magnetism, electromagnetism, the magnetic circuit and the dielectric circuit have been rewritten, using the rationalized mks system. A new chapter on a-c. conversion equipment has been added, and new material has been included in the chapters on transformers, alternators, synchronous motors, and electron tubes and rectifiers.

WELDING PROCESSES AND PROCEDURES, by Joe L. Morris. 255 pages, diagrams, illustrations, 15 × 22 cm. New York, Prentice-Hall, Inc., 1954. Price, $3.75.

A combination of the "how-to" approach for the practical welder and the "why" approach for the engineering student, this little handbook should prove to be a very useful guide. Covering all aspects of welding, and written by an experienced authority, the book is up-to-date and very practical. All techniques and processes of welding are described in detail and are well illustrated with diagrams, photographs and charts.

RADIO RECEIVER DESIGN. PART I, by K. R. Sturley. Second edition, 667 pages, diagrams, 15 × 22 cm. New York, John Wiley & Sons, Inc., 1954. Price, $10.00.

The second edition of Part I of this useful two-part series has not been drastically revised in comparison with the original edition in 1943. In addition to minor revisions for clarity's sake, other changes include: Chapter 1 has been rewritten to emphasize fundamentals of transmission and reception, with noise factor introduced; in Chapter 2 material has been added on valve noise, on wavetraps, signal-to-noise ratio, and balance-to-unbalance aerial feeder connections; and Chapter 7 has been augmented with new material on crystal coupled i.f. transformers.

ENGINEERING FOR AGRICULTURAL DRAINAGE, by Harry Burgess Roe and Quincy Claude Ayres. 501 pages, diagrams, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $7.50.

Farmers, agriculture students and technical men in agricultural work will welcome this thorough, authoritative book on the problems of agricultural drainage. It is the first comprehensive work in this field and as such it brings together information which, up to this time, has been available only in pamphlets, unpublished reports, etc. Because the scope of the book is limited to drainage, all aspects of the subject are covered thoroughly, including soil characteristics, irrigation, moisture control of agricultural peat land and muck, quality control of drain tile and how to protect it against natural enemies (acids, sulfates, etc.), civil law and private rights, pumping, and the approximate cost of various kinds of drainage facilities. The book should become the standard reference work in the field.

COLLEGE ALGEBRA, by Paul K. Rees and Fred W. Sparks. Third edition, 460 pages, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $4.25.

The third edition of this text represents an almost complete revision, for seventeen of the twenty-two chapters have been rewritten, partly to increase clarity and teachability, and partly to provide better illustrative material. All the problems are new, and there are 1000 more than in the previous edition. New material is included on systems of symmetric second degree equations and on systems of homogeneous equations.

THE CLOSED DIE FORGING PROCESS, by P. E. Kyle. 140 pages, illustrations, 14 × 22 cm. New York, The Macmillan Company, 1954. Price, $1.50.

The purpose of this small book is to describe briefly the essential steps in producing drop forgings and to discuss some of the uses to which closed die forgings can be adapted. While not intended as a classroom text, the book will supplement the regular

text in many engineering courses on forging practice. Since closed die forgings are used for high quality parts in such important mechanisms as automobile and aircraft engines, steering mechanisms, power transmission, tractors and others, it is important for today's engineers to understand the processes involved, and the properties of the forged pieces. Here, in one brief volume, are the answers.

ELEMENTARY FLUID MECHANICS, by John K. Vennard. Third edition, 401 pages, diagrams, 15 × 22 cm. New York, John Wiley & Sons, 1954. Price, $5.50.

The author has thoroughly revised this beginning text, increasing the topical coverage by about ten per cent. An introductory chapter on the one dimensional fluid flow has been added; some three hundred new problems have been added and about a hundred and fifty of the former problems have been deleted. Intended for a third year course in engineering, the text requires a knowledge of differential and integral calculus, statics, and dynamics. The revision brings a worthwhile text up-to-date.

APPLIED ELECTRONICS, by Truman S. Gray. Second edition, 881 pages, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc., and The Technology Press (M.I.T.); London, Chapman & Hall, Ltd.; 1954. Price, $9.00.

First published in 1943, this standard work has now been revised to bring the material up-to-date, particularly with reference to the transistor. Retaining, in general, the plan of the first edition, the revised version is intended as a basic course in the applications of electronics—in power, communications, measurement, control, etc. Used as a whole, the text is suitable for a two-semester course; however, it is so arranged that, by using several of the chapters as "outside" reading material, the text may be used in a one semester course emphasizing circuit applications of electron tubes and semiconductor devices.

INORGANIC SYNTHESES. VOLUME IV, edited by John C. Bailar, Jr. 218 pages, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1953. Price, $5.00.

Volume IV in this valuable series on methods of preparing inorganic compounds extends the work of the earlier volumes. Over fifty syntheses are treated, including specific directions for their preparation and precautions to be observed. This latest volume will be especially welcomed by research workers, but it should also prove very helpful for use in graduate courses. The syntheses are arranged according to the Mendeleev periodic classification, with subdivision into A and B groups. Nomenclature is the same as in Volumes II and III, and the index includes all four volumes.

ORGANIC CHEMISTRY: A BRIEF COURSE, by Robert Ward Getchell. 278 pages, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $4.00.

Prepared as a one semester background course in organic chemistry for students majoring in home economics, nursing, agriculture, physical education and the like, this new text is up-to-date, simple, practical and brief. Illustrative material precedes generalizations and laws, and many applications of the principles are included. The text should adequately meet the requirements of the groups to which it is directed.

CURRENT TOPICS

Experimental Irradiation of Foods. —Sterilization of food products may soon be an important commercial peacetime use of atomic energy. For the first time, waste products from atomic bomb manufacture are now to be used for experimental irradiation of foods in the laboratories of the Department of Food Technology at the Massachusetts Institute of Technology.

The Argonne National Laboratories, operated by the University of Chicago in Lemont, Illinois, have announced the shipment to M.I.T. of the first of a new type of experimental radiation source which uses nuclear reactor waste products.

At M.I.T. this source is to be used in connection with equipment which will for the first time make possible irradiating food in commercial-sized containers.

This, says Professor Bernard E. Proctor, head of the M.I.T. Department of Food Technology, brings commercial sterilization of food by radioactivity an important step nearer: the radioactive fission products to be used at M.I.T. are a waste product, presumably inexpensive, and they will be used under conditions which for the first time approximate those needed in commercial food production.

Professor Proctor will be assisted by Dr. Samuel A. Goldblith, Assistant Professor of Food Technology, in the research program which is under the sponsorship of the Atomic Energy Commission through its Division of Reactor Development. It is a continuation of work underway since 1950 in which radioactive cobalt has been used as a source of nuclear energy.

The new energy source now at M.I.T., equivalent in radiation intensity to several pounds of radium, was developed in the Chemical Engineering Division of the Argonne National Laboratory. It is a small hollow cyclindrical block of concrete which was prepared by mixing cement with the liquid radioactive fission products. The latter are the by-products created when neutrons bombard uranium 235 in a nuclear reactor. The highly radioactive cement-fission product mixture is surrounded on all sides by several inches of ordinary concrete and a lead shield which is also several inches thick. This shield is necessary in order that experimenters using the source will not be exposed to the harmful radiation. An opening into the center of the source is provided so that materials to be irradiated may be lowered into the active region.

The source produces high-energy X-rays, technically known as gamma rays, very similar to those from the radioactive cobalt source which has already been studied at M.I.T. But radioactive cobalt is not available in large enough quantities to handle the output of any commercial food producer.

Rays from the new source, Dr. Goldblith says, have a lower average energy level, and the source has a shorter life than the cobalt which has been used before. "But this new source is the material which we will actually have for commercial purposes," says Dr. Goldblith, "so we must learn how to use it."

M.I.T.'s research will be devoted to measurements of the gamma rays produced by the new source, to determining their effects on various foods, and

to learning how long the source can be used without renewal. Special automatic equipment to introduce three small-size food cans into the active region of the new source is now being built to M.I.T. specifications.

Dr. Stephen Lawroski, Director of the Chemical Engineering Division of the Argonne National Laboratory, said that the new source which uses the waste products of a nuclear reactor is regarded as a significant step forward in developing peacetime uses of atomic energy. He claimed three advantages for this type of source: 1. The radioactivity used is obtained as a by-product of reactor operations and as such is inexpensive; 2. It provides, in addition, an ideal method of using the highly radioactive waste products which present serious storage or disposal problems; and 3. It provides for experimental use a radiation source of the type and energy which must be used if large-scale food sterilization by irradiation proves to be feasible.

A Safe, Rolling Step Ladder.—Precision Equipment Co. of Chicago announces production of all steel "automatic action" safety step ladders. A Precision ladder may be easily rolled to the desired position but as soon as a person steps thereon the casters automatically disengage making the ladder immovable. There need be no fear of rolling, swaying or "kick out," since the rubber tipped legs "lock" to the floor, providing a firm, sure base. When a person steps off the ladder the casters instantly and automatically re-engage! The ladder can then be moved easily and quickly.

Precision ladders are designed for use in industrial plants, school shops, garages, stock rooms, warehouses, file departments, bank vaults, libraries, as well as in all types of offices, wherever material is stored above reach. Aluminum-finished steel is used throughout in their construction. Frames are of ¾-in. steel tubing. Reinforced expanded steel makes up the non-slip step plates. The larger four and six step models are furnished with handrails.

New Refining Technique for Germanium.—A new and extremely simple method for refining germanium and other materials to practically perfect purity—99.99999999 per cent pure—has been developed at Bell Telephone Laboratories, research and development organization of the Bell System. Germanium is the material used in transistors.

A substance this pure may very well be the purest material in existence, for it has only one atom of impurity in ten billion atoms of the material. This is about the same as a pinch of salt in 35 freight cars of sugar.

The new process is already in use in the purification of germanium for transistors, the revolutionary new electronic devices invented at Bell Laboratories which can do most of the things done by electron tubes and many other things as well, without some of the handicaps of electron tubes. Actually, the operation of transistors depends on obtaining highly purified germanium.

Zone-melting, as the new purification process is known, was invented by W. G. Pfann, a member of the technical staff of Bell Laboratories.

It is also being used to refine a wide variety of materials other than germanium—metals, other semiconductors and various organic and inorganic substances. Extremely pure tin and antimony, for example, have been prepared by the new process and metallurgists believe many other ultra-pure metallic materials should be available soon.

The new process is based on the fact that impurities are not equally soluble in the solid and liquid states of a substance; usually, impurities are more soluble in the liquid. To take advantage of this, a narrow molten zone is moved slowly along an ingot of relatively impure material to "sweep" the impurities to one end of the ingot.

This is accomplished by passing the ingot through a circular induction heater which, in the case of germanium, brings it to the molten state at a temperature of about 1760 degrees Fahrenheit. As the ingot is passed slowly through the heater, the molten zone tends to hold the impurities while the ingot solidifies into a purer state on the other side of the heater.

In actual practice, a series of such circular heaters is used and each molten zone extracts its share of impurities from that left by the preceding zone. At the end of the run, substantially all the impurities have been "swept" to one end of the ingot. Here they are trapped when the tip of the ingot passes out of the heater and solidifies. This section, loaded with the impurities swept into it from the entire ingot, may then be cut off.

The behavior of germanium in transistors is affected critically by the presence of impurities in the germanium but the kind and amount of impurities must be rigidly controlled for efficient operation. This is accomplished by refining an ingot of germanium to a near-perfect state of purity by the zone-melting process. Then its electrical properties can be altered to the desired degree by the controlled addition of such impurities as arsenic and antimony.

The zone-melting technique, adapted to the requirements of production operation, is now used to make the ultrapure germanium required in the manufacture of transistors by the Western Electric Company, manufacturing and supply unit of the Bell System.

The sweeping action of the zone-melting process serves a dual purpose. In addition to obtaining the utmost in purification, the process also permits the identification of impurities which may be present in such minute amounts that they defy positive detection by all available analytical tools. Once the impurities are concentrated at one end of the ingot, however, they are much more easily identified.

Zone-melting is also being used as a research tool for laboratory investigations of the effects of impurities on various materials, as well as in manufacturing applications.

Butterscotch-Flavored Form of Penicillin.—A new butterscotch-flavored form of potassium penicillin, which requires no refrigeration, is now being distributed by Parke, Davis & Company. The creamy liquid, called Penasoid Suspension, is especially adapted for treating children, but can be prescribed by physicians for oral treatment of most patients in all age groups having diseases caused by penicillin-sensitive organisms. Among the advantages of taking penicillin by mouth are lower incidence of hypersensitivity reactions and greater convenience to patient and physician, the company announcement said.

Penasoid Suspension is supplied in 60-cc. bottles, each cc. containing 60,000 units (300,000 units in a 5-cc. teaspoonful) of crystalline penicillin-G. Dosage is one teaspoonful three or four times daily, or as directed by the physician. The palatable preparation is best absorbed when taken on an empty stomach one to two hours before meals, or two to three hours after meals.

Transducer for Polymer Research.—Invention of a new machine which will be of importance in plastics and colloid research has been announced by a University of Wisconsin chemist. Named the Fitzgerald transducer, the machine was devised by Dr. Edwin R. Fitzgerald working with Prof. John Ferry of the University of Wisconsin department of chemistry for use in their research on polymers—chemicals composed of long strings of molecules and which are the major constituents of natural fibers, gelatinous substances, and plastics. Dr. Fitzgerald, a former project associate at Wisconsin, is now on the faculty of Pennsylvania State University. The transducer was so useful in the work that it became apparent it would be of value for industrial as well as basic research. The transducer is an aid in the study of the structure and mechanical properties of the polymers.

The principle upon which the machine is constructed is that a block of polymer plastic or gel held in place by two parallel plates will resist a vibrating motion partly like a spring and partly like a viscous oil. The resistance can be measured electrically and used to calculate the polymer's properties of viscosity and elasticity.

The reason for the combined spring and oil-like behavior lies in the structure of the chain or rope-like polymer molecules, which are among the longest known to chemists, Prof. Ferry points out. These molecules are in a state of constant whip or snake-like motion at normal temperatures, and as each moves back and forth it alternately opposes and augments the motion of the vibrating plate. From the reaction on the plate, the polymer chemist can make important deductions about molecular structure.

An important feature of the apparatus is the fluid-tight housing for the vibrating plates and electrical coils and equipment which allows complete immersion of the test mechanism in a liquid constant temperature bath. This provides the accurate temperature control necessary for the measurement of mechanical and temperature changes in the plastic under study.

The inventions relating to the new apparatus have been assigned to the Wisconsin Alumni Research Foundation (WARF) which has also arranged for its development and production for industrial purposes by a commercial organization. The first models of the apparatus will be completed soon, the WARF announced.

Perfect Crystals of Pure Iron.—The General Electric Company's top research official has disclosed the development of perfect crystals of pure iron, one-hundred times stronger than any known metallic crystal and inherently resistant to rust.

Explaining crystals as the building blocks that make up metals and alloys, Dr. C. G. Suits, vice president and director of research, said the perfect crystals represent for the first time metals that are as strong as theory predicts they should be and, as such, "provide a new and exciting dimension in metallurgy." He described the crystals as metallic whiskers about one thousandth of an inch thick and an inch or so in length. They were produced in the company's Research Laboratory in Schenectady, New York, by Dr. Robert L. Fullman and Arno Gatti.

"No one knows today how to put these perfect crystals to practical use," he said. "We certainly cannot use them to support a suspension bridge. But their discovery is very recent. In time, applied science and technology will find a practical use for this form of metal."

According to the G-E official, the first observations of metallic whiskers were made at the Bell Telephone Laboratories. Telephone relays under test were found to fail in a mysterious manner, and research showed that whiskers of tin had grown and caused a short circuit.

Steel in machines and copper in electric motors, like all metals and alloys, are aggregates of crystals, so most problems of metals revert to problems of the component crystals, Dr. Suits pointed out.

One of the most important properties of metals and their alloys has to do with strength. He explained that metallurgists have calculated the theoretical strength of iron crystals to be such that one an inch square should support a million pounds or more. It has been a mystery why the strength of actual crystals is a hundred times or so less than this. Metal parts used in machinery and other equipment similarly fall far short of the strength they might theoretically have.

Dr. Suits said that scientists have recently found that crystals previously made, thought to be nearly perfect, actually had atomic irregularities on an atomic scale. These defects caused the weakness.

"Now that we have learned to make some really perfect crystals for the first time, we find that they have astounding strength," he said. "They are stronger than any previously known metal or alloy, and actually attain a tensile strength of nearly a million pounds per square inch."

Moreover, he added, these tiny perfect crystal wires of pure iron do not appear to rust. Finely divided iron, or fine wires of ordinary iron, rusts almost immediately upon exposure to air.

"The same atomic perfection that gives them strength prevents oxidation," he explained.

In order to test the strength of these iron whiskers they are viewed under a microscope as they are bent by means of a micromanipulator. This is a device which makes possible small and accurately controlled movements. From the amount of bending they will withstand, their strength may be calculated.

Belt-Driven Deep Well Pump.—A new pump which can be used on deep wells where normal 60 cycle a-c. electric current is not available, is now being marketed by The F. E. Myers & Bro. Co., Ashland, Ohio. Called the Myers Belt Driven Deep Lift EJECTO, this pump can be operated by virtually any type of electric motor from 1½ to 3 hp., or by a gasoline engine. Vertical motors can be mounted on the pump if desired.

The new pump can be used on wells where the depth to the low water level is as low as 300 ft. Its maximum capacity is 1150 gal. per hour at 100 ft. depth. The pump, which the manufacturer describes as compact and economical, can be off-set at any convenient location from the well, thus eliminating a well pit. With no rods or line shafting to wear, it is also suitable for crooked wells.

Other features of the pump are replaceable bronze wearing ring, a separate bronze vane plate which can be removed for cleaning, and a standardization of parts for all three sizes thus simplifying parts stock.

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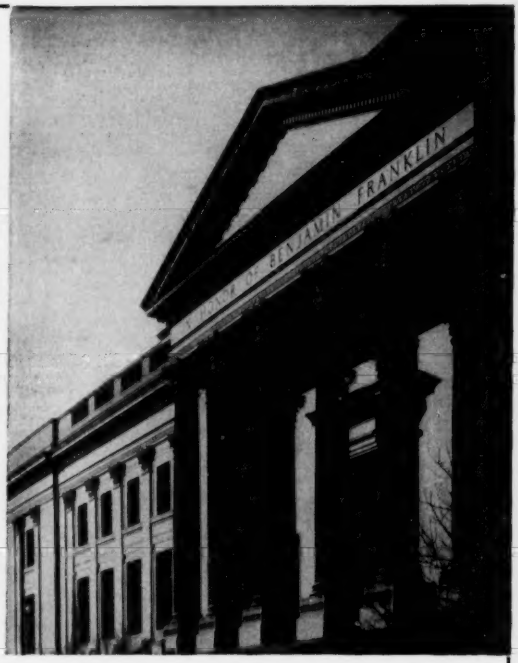

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